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ON THE INTERNAL AND CLOUD-VELOCITIES FOR GROUPS OF STARS OF DIFFERENT TYPES OF SPECTRA

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In his paper, "On Certain Statistical Data Which May Be Valuable in the Classification of the Stars in Order of Their Evolution,"[1] Professor Kapteyn finds, as far as the available data warrant a conclusion, that the linear peculiar velocities of the stars increase with age.[2] This conclusion has been confirmed later from much more extensive data as to radial velocity by Professor Campbell.[3] Professor Kapteyn further points out that this phenomenon very probably entails a gradual dissipation of the star streams with advancing age, and is confirmed in his view by Dyson's[4] observation that the stars of Type I "diverge less from the general drift of the streams than the other stars."

The large extension given to the available number of well-determined astronomical proper motions by the publication of the *Preliminary General Catalogue of 6188 Stars*, by Professor Lewis Boss, and the extreme importance of the two closely connected phenomena renders it exceedingly desirable to test their reality by its aid. To do this is the aim of the present paper.

[1] *Contributions of the Mount Wilson Solar Observatory*, No. 45; *Astrophysical Journal*, **31**, 258, 1910.

[2] This *assumes* that the successive stages of evolution, beginning with the earliest, are in the order of the spectral classes B, A, F, G, K, M.

[3] *Lick Observatory Bulletin*, No. 196, Table II.

[4] *Proceedings of the Roy. Soc. of Edinburgh*, **29**, 390, 1910.

The spectra have been taken, as far as possible, from the catalogues: *Harvard Annals*, **28**, Part 1, Table 12 (Miss Maury); *ibid.*, Part 2, Table 17, and **56**, No. 4, Table 1 (Miss Cannon), and if not contained in these, from *H.R.*, **50**.

I have confined myself provisionally to the stars of spectral classes A and K whose magnitude, according to Boss, is 6.0 or brighter, and all of the stars of the spectral class M. The helium stars are being investigated by Professor Kapteyn himself, and have not been included for that reason.

For the stars of these groups the angles of position of the proper motions have been computed from the data of the *Preliminary General Catalogue*. These angles of position have been used in the way explained below.

Explanation of the method.—Theoretically we have proceeded from the following set of hypotheses, which are identical with those explicitly stated or implied by Mr. Eddington in his paper, "The Systematic Motions of the Stars":[1]

1. The stars are distributed over two drifts of which the centers of gravity are in relative motion.

2. The internal velocities of the stars of each drift are distributed according to Maxwell's law.

3. The modulus, h, is the same for the two drifts.

For the positions of the apex and of the true vertex we have taken for all types of spectra:

$$\begin{array}{rll} \text{Apex} & A = 269^\circ.7, & D = +30^\circ.8 \\ \text{True vertex} & A' = 94^\circ, & D' = +10^\circ \end{array}$$

the equinox being that for 1875.

We will now suppose the sky to be divided into a number of areas and we will call, for the center of any arbitrary area:

α = the right ascension,
δ = the declination,
λ = the distance from the apex,
λ_1 = the distance from the true vertex,
γ = the angle between the direction to the antapex and that to the true vertex taken positively in the sense of a motion from north through west to south.

[1] *Monthly Notices R.A.S.*, **67**, 34, 1906.

The quantities λ, λ_1, and γ may easily be calculated for every area by means of the adopted co-ordinates of the apex and the vertex.

Supposing that only stars of a *determinate class of spectrum* are considered, we will further introduce the following notations:

p = the velocity of the center of gravity of *all* the stars relative to the sun;

g_1 and g_2 = the velocities of the centers of gravity of the first and second drifts, respectively, relative to the center of gravity of all the stars;

g = the velocity of the center of gravity of the first drift relative to that of the second.

Consider a definite small area A of the sky having its center in O. In the tangential plane in O we will fix a rectangular system of co-ordinates OXY, taken in such a way that the sense of a motion directed from the positive end of the axis X to the positive end of the axis of Y is the same as that of a motion from north through west to south. The direction of OX will be left provisionally indeterminate.

We will further put

ω = the angle of position of the direction to the antapex, counted from OX;

ξ_1, η_1, and ξ_2, η_2 = the projections of the velocities of the center of gravity of the first and second stream, respectively, relative to the sun;

h = the modulus entering into the distribution according to Maxwell's law;

u, v = the projections of the internal velocities of an arbitrary star on OX and OY;

N_1, N_2, N_3, N_4 = the number of the stars in A, belonging to the first drift, for which the angle of position of the proper motion counted from OX lies in the first, second, third, and fourth quadrant, respectively;

N = the total number of the stars in A, belonging to the first drift;

n_1, n_2, n_3, n_4, and n = the corresponding quantities for the second drift.

Let A'' represent an area lying diametrically opposite to A and having the same dimensions.

The center O'' is also taken as the origin of a system of co-ordinates $O''X''Y''$ of which the axis $O''X''$ has the same direction as OX, $O''Y''$, a direction opposite to OY.

Further, let ξ_1'', η_1'', ξ_2'', η_2'', N_1'', N_2'' . . . . N'', n_1'', n_2'' . . . . n'' represent for A'' quantities corresponding to those given above for A.

In A as well as in A'' the angles of position of the proper motions are taken positive in the sense north-west-south.

We put further

$$\left.\begin{array}{lll} N_1'=N_1+N_4'' & n_1'=n_1+n_4'' & \mathbf{N}_1=N_1'+n_1' \\ N_2'=N_2+N_3'' & n_2'=n_2+n_3'' & \mathbf{N}_2=N_2'+n_2' \\ N_3'=N_3+N_2'' & n_3'=n_3+n_2'' & \mathbf{N}_3=N_3'+n_3' \\ N_4'=N_4+N_1'' & n_4'=n_4+n_1'' & \mathbf{N}_4=N_4'+n_4' \\ N' =N +N'' & n' =n +n'' & \mathbf{N} =N' +n' \end{array}\right\} \quad (1)$$

Then we have

$$\left.\begin{array}{l} \xi_1=p\sin\lambda\cos\omega+g_1\sin\lambda_1\cos(\omega+\gamma) \\ \eta_1=p\sin\lambda\sin\omega+g_1\sin\lambda_1\sin(\omega+\gamma) \end{array}\right\} \quad (2)$$

Considering further that the angle of position of the observed proper motion lies in the first quadrant if u lies between $-\xi_1$ and ∞ and at the same time v between $-\eta_1$ and ∞, etc., we have

$$\left.\begin{array}{l} N_1=N\dfrac{h^2}{\pi}\displaystyle\int_{-\xi_1}^{+\infty}e^{-h^2u^2}\,du\int_{-\eta_1}^{+\infty}e^{-h^2v^2}\,dv \\ N_2=N\dfrac{h^2}{\pi}\displaystyle\int_{-\infty}^{-\xi_1}e^{-h^2u^2}\,du\int_{-\eta_1}^{+\infty}e^{-h^2v^2}\,dv \\ N_3=N\dfrac{h^2}{\pi}\displaystyle\int_{-\infty}^{-\xi_1}e^{-h^2u^2}\,du\int_{-\infty}^{-\eta_1}e^{-h^2v^2}\,dv \\ N_4=N\dfrac{h^2}{\pi}\displaystyle\int_{-\xi_1}^{+\infty}e^{-h^2u^2}\,du\int_{-\infty}^{-\eta_1}e^{-h^2v^2}\,dv \end{array}\right\} \quad (3)$$

From these we derive easily the following equation:

$$N_1+N_3-(N_2+N_4)=\frac{4N}{\pi}\int_0^{h\xi_1}e^{-t^2}\,dt\int_0^{h\eta_1}e^{-t^2}\,dt. \quad (4)$$

As

$$\xi_1''=\xi_1,\quad \eta_1''=-\eta_1$$

we get in the same way

$$N_1''+N_3''-(N_2''+N_4'')=-\frac{4N''}{\pi}\int_0^{h\xi_1}e^{-t^2}\,dt\int_0^{h\eta_1}e^{-t^2}\,dt. \quad (5)$$

Subtracting (5) from (4), we get for the first drift

$$N_1'+N_3'-(N_2'+N_4')=\frac{4N'}{\pi}\int_0^{h\xi_2} e^{-p^2}\,dt\int_0^{h\eta_2} e^{-p^2}\,dt. \tag{6}$$

For the second drift, we get in the same way

$$\left.\begin{aligned}\xi_2&=p\sin\lambda\cos\omega-g_2\sin\lambda_1\cos(\omega+\gamma),\\ \eta_2&=p\sin\lambda\cos\omega-g_2\sin\lambda_1\sin(\omega+\gamma),\end{aligned}\right\} \tag{7}$$

$$n_1'+n_3'-(n_2'+n_4')=\frac{4n'}{\pi}\int_0^{h\xi_2} e^{-p^2}\,dt\int_0^{h\eta_2} e^{-p^2}\,dt. \tag{8}$$

Therefore, taking the two drifts together,

$$\left.\begin{aligned}\mathbf{N}_1+\mathbf{N}_3-(\mathbf{N}_2+\mathbf{N}_4)=\frac{4N'}{\pi}\int_0^{h\xi_2} e^{-p^2}\,dt\int_0^{h\eta_2} e^{-p^2}\,dt+\\ \frac{4n'}{\pi}\int_0^{h\xi_2} e^{-p^2}\,dt\int_0^{h\eta_2} e^{-p^2}\,dt.\end{aligned}\right\} \tag{9}$$

Hough and Halm have shown that the mixture of the two drifts is not constant for the whole sky.[1] According to these authors the law of proportion for the different parts of the sky may be tolerably accounted for by an equation of the form

$$\frac{n}{N+n}=s_0+\epsilon\cos\beta \tag{10}$$

s_0 and ϵ being constants, β the distance from the center of the area considered to a point where the maximum density of the second-drift stars occurs. We may write this

$$N-n=(N+n)a+(N+n)C\cos\beta \tag{11}$$

for the area A, and

$$N''-n''=(N''+n'')a-(N''+n'')C\cos\beta \tag{12}$$

for the area A'', a and C being constant for the whole sky.

Therefore by addition

$$N'-n'=(N'+n')a+\{N+n-(N''+n'')\}C\cos\beta. \tag{13}$$

[1] "A Spectroscopic Determination of the Systematic Motion of the Stars," *Monthly Notices R.A.S.*, **70**, 85, 1909; "On the Systematic Motions of the Bradley Stars," *ibid.*, **70**, 568, 1910.

If now we assume that the distribution of our stars over the sky is such that diametrical areas A and A'' contain equal total numbers of stars, a condition that is approximately fulfilled in our case, we have

$$N+n=N''+n''. \tag{14}$$

Therefore, according to (13), n'/N' will be constant. We will adopt

$$\frac{n'}{N'}=0.68, \tag{15}$$

the average value found by Eddington[1] from the stars of the *Preliminary General Catalogue*.

We thus get

$$N'=0.596\mathbf{N}, \qquad n'=0.404\mathbf{N}. \tag{16}$$

We have further as a consequence of the definitions of g_1 and g_2,

$$N'g_1=n'g_2. \tag{17}$$

Therefore

$$\left.\begin{aligned} hg_1&=0.404h(g_1+g_2)=0.404hg,\\ hg_2&=0.596h(g_1+g_2)=0.596hg. \end{aligned}\right\} \tag{18}$$

We may still dispose arbitrarily of the directions of the axes OX of the systems of co-ordinates in the areas A. We will choose these in two different ways, first in such a way that for every region A the angle of position of the direction to the true vertex counted from OX is 45°.

In this case

$$\omega=45^\circ-\gamma. \tag{19}$$

The quantities $\mathbf{N}_1$, ... etc., $\mathbf{N}$ are completely determined by the empirical data and may easily be derived for every pair of diametrical areas.

From (2), (7), (18), and (19) we derive

$$\left.\begin{aligned} h\xi_1&=hp\sin\lambda\cos(45^\circ-\gamma)+0.286hg\sin\lambda_1\\ h\eta_1&=hp\sin\lambda\sin(45^\circ-\gamma)+0.286hg\sin\lambda_1\\ h\xi_2&=hp\sin\lambda\cos(45^\circ-\gamma)-0.421hg\sin\lambda_1\\ h\eta_2&=hp\sin\lambda\sin(45^\circ-\gamma)-0.421hg\sin\lambda_1 \end{aligned}\right\} \tag{20}$$

Furthermore, from (9) and (16)

[1] "The Systematic Motions of the Stars of Professor Boss's Preliminary General Catalogue," *Monthly Notices R.A.S.*, **71**, 38, 1910.

$$\mathbf{N}_1+\mathbf{N}_3-(\mathbf{N}_2+\mathbf{N}_4)=\frac{4\mathbf{N}}{\pi}\left\{\begin{array}{l} 0.596\int_0^{h\xi_1} e^{-p}\,dt\int_0^{h\eta_1} e^{-p}\,dt+ \\ 0.404\int_0^{h\xi_2} e^{-p}\,dt\int_0^{h\eta_2} e^{-p}\,dt\Big\}. \end{array}\right\} \quad (21)$$

The quantities λ, λ_1, and γ having been computed for every area A by means of the adopted positions of apex and vertex, the quantities $h\xi_1$, $h\eta_1$, $h\xi_2$, and $h\eta_2$ will be determined by the equations (20) in function of the two unknown quantities hp and hg, and the second member of equation (21) will also depend on no other unknown quantities than these two. Taking now all areas together we get a set of equations between the quantities hp and hg that will be especially favorable for the determination of hg, but which will yield a value of hp, the weight of which is very small indeed. In order to obtain a more reliable determination of this quantity we will derive another set of equations.

For that purpose we will now choose our system of co-ordinates in A in another way and let OX coincide with the direction to the antapex.

We have then

$$\omega=0 \quad (22)$$

and from (2) and (7), with the aid of (18)

$$\left.\begin{array}{l} h\xi_1=hp\sin\lambda+0.404hg\sin\lambda_1\cos\gamma, \\ h\xi_2=hp\sin\lambda-0.596hg\sin\lambda_1\cos\gamma. \end{array}\right\} \quad (23)$$

From equations (3) we derive for an arbitrary area A

$$N_1+N_4-(N_2+N_3)=2\frac{N}{\sqrt{\pi}}\int_0^{h\xi_1} e^{-p}\,dt. \quad (24)$$

By combining this equation with the equivalent one for A'', by the aid of (16) we get

$$N_1'+N_4'-(N_2'+N_3')=\frac{2\mathbf{N}}{\sqrt{\pi}}0.596\int_0^{h\xi_1} e^{-p}\,dt, \quad (25)$$

or combining again with the equivalent one for the second drift

$$\mathbf{N}_1+\mathbf{N}_4-(\mathbf{N}_2+\mathbf{N}_3)=\frac{2\mathbf{N}}{\sqrt{\pi}}\Big\{0.596\int_0^{h\xi_1} e^{-p}\,dt+0.404\int_0^{h\xi_2} e^{-p}\,dt\Big\}. \quad (26)$$

In this equation we have of course to take for $\mathbf{N}_1 \ldots \mathbf{N}_4$ the quantities corresponding to the *new* direction of OX. Taking

now all areas together we get a second set of equations which combined with (21) enables us to derive the quantities hp and hg by trial. It will be convenient to combine the equations (21) and (26) into a single equation for the whole sky, taking care of course that every pair of diametrical areas shall get its proper weight. In following up this idea we have chosen the weights in such a way that the first and second members of equations (26) were multiplied by sin λ, those of equations (21) by $\sin^2 \lambda_1 \cdot e^{-h^2p^2 \sin^2 \lambda}$, adopting in the latter expression for hp a preliminary approximate value.

For the sake of clearness we sum up what has been said about the method which will be applied below, as follows:

We divide the sky into a number of areas which are by pairs equal and diametrically opposite. For every pair A and A'' we derive the quantities $\mathbf{N}$ and further the quantities $\mathbf{N}_1$, $\mathbf{N}_2$, $\mathbf{N}_3$, $\mathbf{N}_4$ in two different ways, viz.:

1. Taking in every area A for OX the direction to the antapex;

2. Taking in every area A for OX such a direction that the angle of position of the direction to the true vertex of the first drift becomes 45°.

Corresponding to these two cases we have then the follo wing sets of equations:

CASE 1

$$\left.\begin{aligned} h\xi_1 &= hp \sin \lambda + 0.404 hg \sin \lambda_1 \cos \gamma \\ h\xi_2 &= hp \sin \lambda - 0.596 hg \sin \lambda_1 \cos \gamma \\ \frac{2}{\sqrt{\pi}} \sum \mathbf{N} \sin \lambda \Big\{ 0.596 \int_0^{h\xi_1} e^{-t^2} dt + 0.404 \int_0^{h\xi_2} e^{-t^2} dt \Big\} &= \\ \Sigma \sin \lambda \{ (\mathbf{N}_1 + \mathbf{N}_4) - (\mathbf{N}_2 + \mathbf{N}_3) \}. & \end{aligned}\right\} \quad (27)$$

CASE 2

$$\left.\begin{aligned} h\xi_1 &= hp \sin \lambda \cos (45° - \gamma) + 0.286 hg \sin \lambda_1 \\ h\eta_1 &= hp \sin \lambda \sin (45° - \gamma) + 0.286 hg \sin \lambda_1 \\ h\xi_2 &= hp \sin \lambda \cos (45° - \gamma) - 0.421 hg \sin \lambda_1 \\ h\eta_2 &= hp \sin \lambda \sin (45° - \gamma) - 0.421 hg \sin \lambda_1 \\ \frac{4}{\pi} \sum \mathbf{N} \sin^2 \lambda_1 e^{-h^2p^2 \sin^2 \lambda} \Big\{ 0.596 \int_0^{h\xi_1} e^{-t^2} dt \int_0^{h\eta_1} e^{-t^2} dt + & \\ 0.404 \int_0^{h\xi} e^{-t^2} dt \int_0^{h\eta_2} e^{-t^2} dt = & \\ \Sigma \sin^2 \lambda_1 e^{-h^2p^2 \sin^2 \lambda} \{ (\mathbf{N}_1 + \mathbf{N}_3) - (\mathbf{N}_2 + \mathbf{N}_4) \} & \end{aligned}\right\} \quad (28)$$

The quantities *hp* and *hg* have then to be solved by trial and error.

Applications of the method.—In applying the method explained above first to the A stars and subsequently to the K and M stars of the *Preliminary General Catalogue*, we have excluded those stars of which it is known that they belong to local drifts. We thus excluded 23 stars belonging to the group of the *Hyades* and furthermore the stars β, γ, δ, ϵ, and ζ *Ursae Majoris*.

The sky has been divided into the following areas, according to the values of Ω and λ, Ω being the angle Vertex-Apex-Star:

TABLE I

Limits of Ω	Limits of λ	Limits of Ω	Limits of λ
A 0° and 360°	0° and 30°	*H* 270° and 315°	30° and 70°
B 0° " 45°	30° " 70°	*I* 315° " 360°	30° " 70°
C 45° " 90°	30° " 70°	*K* 0° " 45°	70° " 110°
D 90° " 135°	30° " 70°	*L* 45° " 90°	70° " 110°
E 135° " 180°	30° " 70°	*M* 90° " 135°	70° " 110°
F 180° " 225°	30° " 70°	*N* 135° " 180°	70° " 110°
G 225° " 270°	30° " 70°		

and further the corresponding diametrical areas A'', B'', C'', etc., together 26 areas. For the centers of the areas *A*, *B*, *C*, etc., we have adopted points not coinciding precisely with the geometrical centers of gravity, but points obtained in the following way. Leaving the co-ordinates of the stars of *A*, *B*, *C*, etc., unaltered, the positions of the stars A'', B'', C'', etc., have been transferred diametrically to *A*, *B*, *C*, etc. By combining now the positions of the stars of *A*, *B*, *C*, etc., with the transferred positions of the stars of A'', B'', C'', etc., we get systems of fictitious positions of stars. The centers of gravity of these systems have been adopted as the centers of the areas, *A*, *B*, *C*, etc. With the co-ordinates of these centers as arguments, the quantities λ, λ_1, and γ corresponding to the different pairs of areas have been calculated. Of course this has been done for the different types of spectra separately.

In combining the equations (27) the areas *A* and A'', which should have hardly any weight and would, moreover, cause some difficulty, have been simply neglected. In order to use them in combining the equations (28), they have been subdivided again, each into four areas according to the following scheme:

TABLE II

sin λ cos (45°−γ)	sin λ sin (45°−γ)	sin λ cos (45°−γ)	sin λ sin (45°−γ)
A_1 +	+	A_1'' +	+
A_2 +	−	A_2'' −	+
A_3 −	+	A_3'' +	−
A_4 −	−	A_4'' −	−

the areas A_1'', A_2'', A_3'', and A_4'' lying evidently again diametrically to A_1, A_2, A_3, and A_4, respectively.

The results.—In the Tables III to VII the main results from the different groups of stars have been condensed. At the head of each table have been given: the designation of the group considered, the total number of stars included in the group, and the resulting values of *hp* and *hg* with their *mean* errors. In the tables themselves we have used for the sake of brevity the following notation. First, every pair of combined diametrical areas has been denoted by one letter, the combination of A_1 and A_1'' being indicated by A_1', of B and B'' by B', etc. This notation has been given in the first column.

Further, we have put

$$\left.\begin{aligned}
P'_{\text{obs.}} &= (\mathbf{N}_1+\mathbf{N}_4)-(\mathbf{N}_2+\mathbf{N}_3)\\
P'_{\text{comp.}} &= \frac{2}{\sqrt{\pi}}\mathbf{N}\left\{0.596\int_0^{h\xi_1} e^{-t^2}dt+0.404\int_0^{h\xi_2} e^{-t^2}dt\right\}\\
O-C &= P'_{\text{obs.}}-P'_{\text{comp.}}\\
Q'_{\text{obs.}} &= (\mathbf{N}_1+\mathbf{N}_3)-(\mathbf{N}_2+\mathbf{N}_4)\\
Q'_{\text{comp.}} &= \frac{4}{\pi}\mathbf{N}\left\{0.596\int_0^{h\xi_1} e^{-t^2}dt\int_0^{h\eta_1} e^{-t^2}dt+\right.\\
&\qquad \left. 0.404\int_0^{h\xi_2} e^{-t^2}dt\int_0^{h\eta_2} e^{-t^2}dt\right\}\\
O_1-C_1 &= Q'_{\text{obs.}}-Q'_{\text{comp.}}
\end{aligned}\right\} \quad (29)$$

The amounts of these quantities have been given in the columns 2, 3 . . . . 7.

The values of $P'_{\text{comp.}}$ and $Q'_{\text{comp.}}$ have been computed for every group of stars by means of the values of *hp* and *hg* at the head of the corresponding table. Further, in deriving the quantities $P'_{\text{obs.}}$, $P'_{\text{comp.}}$, $Q'_{\text{obs.}}$, and $Q'_{\text{comp.}}$ the directions of OX have been taken in accordance with the systems denoted above under Case 1 and Case 2, respectively.

The eighth column contains the numbers **N** defined above. In order to reduce the residuals $O-C$ and O_1-C_1 for the different pairs of areas to quantities of the same weight, they have been divided by $\sqrt{\mathbf{N}}$. The quantities obtained in this way are given in the ninth and tenth columns.

TABLE III

A STARS (NO AREAS EXCLUDED), 1089 STARS

$hp=1.16$, $m.e.=\pm 0.09$; $hg=2.28$, $m.e.=\pm 0.32$

P' obs.	P' (comp.)	$O-C$	Q' obs.	Q' (comp.)	O_1-C_1	N	$\frac{O-C}{\sqrt{N}}$	$\frac{O_1-C_1}{\sqrt{N}}$
A_1'	.....	.....	+20	+25.7	− 5.7	55	.....	−0.77
A_2'	.....	.....	+15	+13.8	+ 1.2	68	.....	+0.15
A_3'	.....	.....	+ 9	+ 7.0	+ 2.0	42	.....	+0.31
A_4'	.....	.....	+12	+ 4.1	+ 7.9	26	.....	+1.55
B' +59	+42.8	+16.2	+46	+51.4	− 5.4	98	+1.64	−0.55
C' +67	+45.5	+21.5	+ 4	+37.7	−33.7	90	+2.27	−3.55
D' +41	+37.6	+ 3.4	+21	+ 7.2	+13.8	55	+0.46	+1.86
E' +57	+61.7	− 4.7	− 1	− 2.5	+ 1.5	81	−0.52	+0.17
F' +58	+61.4	− 3.4	+26	+ 0.2	+25.8	80	−0.38	+2.89
G' +37	+45.4	− 8.4	+15	+ 7.6	+ 7.4	65	−1.04	+0.92
H' +22	+20.5	+ 1.5	+18	+14.6	+ 3.4	38	+0.24	+0.55
J' +24	+22.4	+ 1.6	+34	+28.6	+ 5.4	54	+0.22	+0.73
K' +74	+71.0	+ 3.0	+68	+48.9	+19.1	108	+0.29	+1.84
L' +43	+49.0	− 6.0	+29	+25.6	+ 3.4	75	−0.69	+0.39
M' +34	+45.6	−11.6	+30	+23.9	+ 6.1	70	−1.39	+0.73
N' +48	+55.5	− 7.5	+36	+38.2	− 2.2	84	−0.82	−0.24

TABLE IV

A STARS (AREAS C' AND F' EXCLUDED), 919 STARS

$hp=1.22$, $m.e.=\pm 0.09$; $hg=2.58$, $m.e.=\pm 0.21$

P' obs.	P' (comp.)	$O-C$	Q' obs.	Q' (comp.)	O_1-C_1	N	$\frac{O-C}{\sqrt{N}}$	$\frac{O_1-C_1}{\sqrt{N}}$
A_1'	.....	.....	+20	+30.0	−10.0	55	.....	−1.35
A_2'	.....	.....	+15	+17.7	− 2.7	68	.....	−0.33
A_3'	.....	.....	+ 9	+ 9.1	− 0.1	42	.....	−0.02
A_4'	.....	.....	+12	+ 4.8	+ 7.2	26	.....	+1.41
B' +59	+38.6	+20.4	+46	+56.9	−10.9	98	+2.06	−1.10
C'	.....	.....	....	.....	.....	..	.....	.....
D' +41	+37.7	+ 3.3	+21	+ 9.8	+11.2	55	+0.44	+1.51
E' +57	+63.2	− 6.2	− 1	− 2.2	+ 1.2	81	−0.69	+0.13
F'	.....	.....	....	.....	.....	..	.....	.....
G' +37	+45.4	− 8.4	+15	+10.5	+ 4.5	65	−1.04	+0.56
H' +22	+19.3	+ 2.7	+18	+17.2	+ 0.8	38	+0.44	+0.13
J' +24	+20.2	+ 3.8	+34	+31.7	+ 2.3	54	+0.52	+0.31
K' +74	+67.8	+ 6.2	+68	+51.3	+16.7	108	+0.60	+1.61
L' +43	+46.8	− 3.8	+29	+29.1	− 0.1	75	−0.44	−0.01
M' +34	+43.4	− 9.4	+30	+27.4	+ 2.6	70	−1.12	+0.31
N' +48	+53.1	− 5.1	+36	+40.1	− 4.1	84	−0.56	−0.45

TABLE V

K STARS, 1097 STARS

$hp=0.69$, $m.e.=\pm 0.05$; $hg=1.05$, $m.e.=\pm 0.14$

P' obs.	P' (comp.)	$O-C$	Q' obs.	Q' (comp.)	O_1-C_1	N	$\frac{O-C}{\sqrt{N}}$	$\frac{O_1-C_1}{\sqrt{N}}$
A_1'	.....	.....	+ 1	+ 8.2	− 7.2	62	.....	−0.91
A_2'	.....	.....	+ 9	+ 1.9	+ 7.1	49	.....	+1.01
A_3'	.....	.....	− 1	+ 1.3	− 2.3	36	.....	−0.38
A_4'	.....	.....	+ 7	+ 1.3	+ 5.7	27	.....	+1.10
B' +44	+52.3	− 8.3	+28	+26.9	+ 1.1	112	−0.78	+0.10
C' +46	+40.5	+ 5.5	+10	+12.1	− 2.1	84	+0.60	−0.23
D' +41	+32.6	+ 8.4	+ 7	+ 1.2	+ 5.8	63	+1.06	+0.73
E' +56	+39.8	+16.2	− 3	+ 1.1	− 4.1	73	+1.90	−0.48
F' +45	+38.2	+ 6.8	+ 5	+ 2.8	+ 2.2	69	+0.82	+0.26
G' +30	+31.1	− 1.1	+ 1	+ 1.7	− 0.7	59	−0.14	−0.09
H' +23	+27.7	− 4.7	+17	+ 8.0	+ 9.0	57	−0.62	+1.19
J' +17	+27.1	−10.1	+ 6	+14.4	− 8.4	58	−1.33	−1.10
K' +55	+59.0	− 4.0	+23	+25.0	− 2.0	99	−0.40	−0.20
L' +50	+44.2	+ 5.8	+20	+11.2	+ 8.8	74	+0.67	+1.02
M' +39	+50.5	−11.5	+ 7	+12.7	− 5.7	85	−1.25	−0.62
N' +54	+53.6	+ 0.4	+20	+21.5	− 1.5	90	+0.04	−0.16

TABLE VI

M STARS, 240 STARS

$hp=0.76$, $m.e.=\pm 0.06$; $hg=1.38$, $m.e.=\pm 0.39$

P' obs.	P' (comp.)	$O-C$	Q' obs.	Q' (comp.)	O_1-C_1	N	$\frac{O-C}{\sqrt{N}}$	$\frac{O_1-C_1}{\sqrt{N}}$
A_1'	.....	.....	+ 3	+ 2.2	+ 0.8	11	.....	+0.24
A_2'	.....	.....	+ 7	+ 0.9	+ 6.1	9	.....	+2.03
A_3'	.....	.....	+ 3	+ 0.7	+ 2.3	9	.....	+0.77
A_4'	.....	.....	− 2	+ 0.3	− 2.3	4	.....	−1.15
B' +16	+14.8	+ 1.2	+12	+10.0	+ 2.0	32	+0.21	+0.35
C' + 7	+ 5.5	+ 1.5	+ 5	+ 2.5	+ 2.5	11	+0.45	+0.75
D' + 7	+ 6.1	+ 0.9	− 1	+ 1.1	− 2.1	11	+0.27	−0.63
E' + 7	+ 5.5	+ 1.5	+ 2	+ 0.5	+ 1.5	9	+0.50	+0.50
F' +10	+10.6	− 0.6	− 1	+ 1.2	− 2.2	18	−0.14	−0.52
G' + 5	+ 5.9	− 0.9	− 1	+ 0.7	− 1.7	11	−0.27	−0.51
H' + 3	+ 4.4	− 1.4	− 1	+ 1.8	− 2.8	9	−0.47	−0.93
J' +12	+10.4	+ 1.6	+12	+ 7.8	+ 4.2	24	+0.33	+0.86
K' + 7	+ 7.8	− 0.8	+ 9	+ 4.0	+ 5.0	13	−0.22	+1.39
L' +12	+13.2	− 1.2	− 2	+ 4.5	− 6.5	22	−0.26	−1.39
M' +22	+18.1	+ 3.9	0	+ 6.1	− 6.1	30	+0.71	−1.11
N' + 5	+10.0	− 5.0	+ 3	+ 5.2	− 2.2	17	−1.21	−0.53

In the first solution of the A stars the pairs of areas C' and F' yield somewhat large values for the residual quantities $\frac{O_1-C_1}{\sqrt{N}}$, the former a negative, the latter a positive one (see Table III). Now the factor $\sin^2\lambda_1 e^{-h^2p^2\sin^2\lambda}$ with which the terms given by the

TABLE VII

K AND M STARS TOGETHER, 1337 STARS

$hp = 0.70$, *m.e.* $= \pm 0.05$; $hg = 1.11$, *m.e.* $= \pm 0.12$

P' obs.	P' (comp.)	$O-C$	Q' obs.	Q' (comp.)	O_1-C_1	N	$\frac{O-C}{\sqrt{N}}$	$\frac{O_1-C_1}{\sqrt{N}}$
A_1'	.....	.....	+ 4	+10.5	− 6.5	73	.....	−0.76
A_2'	.....	.....	+16	+ 2.8	+13.2	58	.....	+1.73
A_3'	.....	.....	+ 2	+ 1.8	+ 0.2	45	.....	+0.03
A_4'	.....	.....	+ 5	+ 1.6	+ 3.4	31	.....	+0.61
B' +60	+67.0	− 7.0	+40	+36.4	+ 3.6	144	−0.58	+0.30
C' +53	+45.9	+ 7.1	+15	+15.0	0.0	95	+0.73	0.00
D' +48	+38.7	+ 9.3	+ 6	+ 2.4	+ 3.6	74	+1.08	+0.42
E' +63	+45.4	+17.6	− 1	+ 1.6	− 2.6	82	+1.94	−0.29
F' +55	+48.5	+ 6.5	+ 4	+ 3.9	+ 0.1	87	+0.70	+0.01
G' +35	+37.0	− 2.0	0	+ 2.4	− 2.4	70	−0.24	−0.29
H' +26	+32.1	− 6.1	+16	+10.0	+ 6.0	66	−0.75	+0.74
J' +29	+37.7	− 8.7	+18	+21.2	− 3.2	82	−0.96	−0.35
K' +62	+66.6	− 4.6	+32	+29.3	+ 2.7	112	−0.43	+0.26
L' +62	+57.3	+ 4.7	+18	+15.5	+ 2.5	96	+0.48	+0.26
M' +51	+68.3	− 7.3	+ 7	+18.3	−11.3	115	−0.68	−1.05
N' +59	+63.7	− 4.7	+23	+26.8	− 3.8	107	−0.45	−0.37

different pairs of areas enter into the equations (28) happens to be large for C', and small for F'. The part which these pairs of areas have played in the calculation of *hg* must therefore be rather large for C', and small for F'. As a consequence we see that the residuals of the other pairs of areas show some preference for positive values. It seems therefore probable that the value for *hg* obtained in the first solution will be somewhat too small as a consequence of some anomaly in the distribution of the proper motions in C'. The evidence for the reality of an abnormal character of C' is increased by the fact that one of the two diametrical areas of C' lies in *Cygnus* near a region of the sky which also appeared to behave anomalously in the researches of Mr. Eddington.[1] For this reason we have made a second solution for the A stars excluding now C' and F' (see Table IV). The values for *hp* and *hg* resulting from this solution will probably be better than those from the first solution. Owing to the small number of M stars, the results for this class are necessarily somewhat uncertain. We have made for that reason another solution based on the K and M stars combined. The results of this solution have been given in Table VII.

[1] Region $G\gamma$ in *Monthly Notices R.A.S.*, **57**, 52, 1896; region VII in *ibid.*, **71**, 37, 1910.

If now we compare the values of *hg* derived from the different groups of stars, we see that this quantity decreases by a considerable amount if we pass from the A to the K stars. Adopting for the A stars the value of *hg* from the first solution, we get a difference of 1.23 ± 0.35.[1] Adopting the value from the second solution we get 1.53 ± 0.25. Passing now from the K stars to the M stars we get again some increase of the value for *hg*. But we get only a difference of 0.33 ± 0.42, which may therefore be accidental. Finally, taking the K and M stars together, we get

hg for A stars (first solution) $-$ *hg* for K and M stars $= 1.17 \pm 0.34$
hg for A stars (second solution) $-$ *hg* for K and M stars $= 1.47 \pm 0.24$

Adopting for the A stars the value of *hg* from the first solution we get for the *difference* a value 3.4 times larger than the mean error, and adopting the value from the second solution we get a value 6.1 times larger than the mean error. The difference cannot well be accidental, therefore.

Denoting the mean internal velocity of the stars of a drift by Ω, and considering that we have

$$h = \frac{2}{\Omega \sqrt{\pi}},$$

we get:

$$\left.\begin{aligned} &\text{A stars (first solution)} : \frac{g}{\Omega} = 2.02 \pm 0.28 \\ &\text{A stars (second solution)}: \frac{g}{\Omega} = 2.29 \pm 0.19 \\ &\text{K and M stars together} : \frac{g}{\Omega} = 0.98 \pm 0.11 \end{aligned}\right\} \quad (30)$$

We see, therefore, that the second of the hypotheses mentioned at the head of this paper is clearly confirmed by our results. For, taking in every group of stars the mean internal velocity of the stars of the group as unity, the relative velocity of the two drifts expressed in this unit *decreases* with age. Or, stating the result in another way: if for every spectral class we take the relative velocity of the two drifts as unity, the mean internal velocity will *increase* with age. In other words, *the phenomenon of star-streaming*

[1] We have used throughout the *mean* error, not the probable error.

shows itself in greater purity for the younger spectral types than for the older ones, which is in conformity with what Professor Kapteyn's paper led us to expect.

Of course the results hitherto considered do not enable us to decide whether the effect stated here is caused by an increase of the *absolute* value of the mean internal velocity, or by a decrease in the relative velocity of the two drifts; or, perhaps, by the two causes together. We may, however, express all the velocities in function of what is commonly called the sun's velocity (p). From Tables III, IV, and VII we get:

$$\left.\begin{aligned} &\text{A stars (first solution)} \quad : \frac{p}{\Omega} = 1.03 \pm 0.08 \\ &\text{A stars (second solution)}: \frac{p}{\Omega} = 1.08 \pm 0.08 \\ &\text{K and M stars} \quad : \frac{p}{\Omega} = 0.62 \pm 0.04 \end{aligned}\right\} \quad (31)$$

or, combining the values for hp given by these tables with the corresponding values for hg,

$$\left.\begin{aligned} &\text{A stars (first solution)} \quad : \frac{g}{p} = 1.97 \pm 0.31 \\ &\text{A stars (second solution)}: \frac{g}{p} = 2.11 \pm 0.23 \\ &\text{K and M stars} \quad : \frac{g}{p} = 1.59 \pm 0.20 \end{aligned}\right\} \quad (32)$$

If it were allowable to assume that the motion of the sun were the same relatively to the center of gravity of the K and M stars as it is relatively to that of the A stars, it would be easy to draw a conclusion. For in this case the quantity p would be equal for the two groups. Therefore the results of (31) would then prove clearly that the quantity Ω increases with age, while those of (32) would yield some evidence for a decrease of g with age, the latter being somewhat doubtful as a consequence of the relatively large amounts of the mean error.

Recent unpublished results obtained by Professor Kapteyn make it doubtful, however, whether the assumption mentioned is permissible. As a consequence we cannot, from the results obtained above, draw safe conclusions about the quantities g

and Ω separately, without taking into consideration supplementary data. In order to meet the difficulty I have tried to derive the values of Ω in kilometers from spectroscopic velocities. The data I have used for that purpose have been taken from a list compiled by Professor Kapteyn mainly from spectroscopic velocities published by the observatories of Bonn and Yerkes, velocities of the centers of gravity of spectroscopic binaries published by Campbell, and some unpublished spectroscopic velocities derived at the Yerkes Observatory. By means of an examination of the corresponding astronomical proper motions, as given in the *Preliminary General Catalogue*, 12 stars from the list, belonging to spectral class A, and 27 belonging to the classes K or M could be shown with very great probability to belong to the first drift. From the spectroscopic velocities of these stars the velocity V_1 of the center of gravity of the first drift relative to the sun has been calculated separately for the A and for the K and M stars. In doing so I have adopted for the co-ordinates a_1 and δ_1 of the apparent vertex of the first drift quantities chosen in such a way that they correspond with the adopted positions for the apex and the true vertex and with the values for g/p found above, viz.:

$$\left.\begin{array}{ll} \text{for the A stars,} & a_1 = 91^\circ.8, \quad \delta_1 = -12^\circ.5 \\ \text{for the K and M stars,} & a_1 = 91^\circ.5, \quad \delta_1 = -15^\circ.1. \end{array}\right\} \quad (33)$$

I now made two solutions for every group: (*a*) assuming that the observed radial velocities for stars of a determined spectral class need a constant correction (for pressure-shift ?); and (*b*) neglecting such a term. I found:

$$\left.\begin{array}{ll} \text{A stars, solution } (a), & V_1 = 19.8 \text{ km} \\ \text{A stars, solution } (b), & V_1 = 21.1 \text{ km} \\ \text{K and M stars, solution } (a), & V_1 = 23.1 \text{ km} \\ \text{K and M stars, solution } (b), & V_1 = 20.8 \text{ km} \end{array}\right\} \quad (34)$$

From the residuals obtained by substituting these values for V_1 in the equations of condition, I derived the values for Ω corresponding to the four solutions of V_1 by multiplying the mean of the residuals (every residual taken with the positive sign) by *two*. I thus found:

$$\left.\begin{array}{ll} \text{A stars, solution } (a), & \Omega = 13.4 \text{ km} \\ \text{A stars, solution } (b), & \Omega = 15.3 \text{ km} \\ \text{K and M stars, solution } (a), & \Omega = 24.6 \text{ km} \\ \text{K and M stars, solution } (b), & \Omega = 24.9 \text{ km} \end{array}\right\} \quad (35)$$

We see that the values for Ω resulting from the K and M stars are considerably larger than the corresponding values for the A stars. We will adopt:

$$\left.\begin{array}{ll} \text{A stars,} & \Omega = 14.3 \text{ km (12 stars)} \\ \text{K and M stars,} & \Omega = 24.7 \text{ km (27 stars).} \end{array}\right\} \quad (36)$$

By combining these results with those of (30) and (31), we find:

$$\left.\begin{array}{ll} \text{A stars, first solution,} & p = 14.7 \text{ km; } g = 28.9 \text{ km} \\ \text{A stars, second solution,} & p = 15.4 \text{ km; } g = 32.7 \text{ km} \\ \text{K and M stars,} & p = 15.3 \text{ km; } g = 24.2 \text{ km} \end{array}\right\} \quad (37)$$

The results of (36) and (37) seem to show that the decrease of $\frac{g}{\Omega}$ with age, stated above, is due in larger part to an increase of Ω, and in less degree to a decrease of g.

Meanwhile, owing to the small number of radial velocities, our results in the form (36) and (37) are quite unreliable. We may, however, confidently expect better results in one or two years. For even now, if only existing observations were published, the data would probably be sufficient for the derivation of fairly trustworthy results. Moreover, observations are now in progress on Mount Wilson for the express purpose of furnishing the data required for investigations closely related to the present one.

ASTRONOMICAL LABORATORY
GRONINGEN
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ON THE DISTRIBUTION OF BRIGHTNESS IN THE TAIL OF HALLEY'S COMET[1]

By K. SCHWARZSCHILD and E. KRON

The expedition of the Potsdam Astrophysical Observatory, which went to Teneriffe in the spring of 1910, in charge of Professor Müller, took with it for photographing Halley's comet only a small stereoscopic camera in which either two Zeiss aplanatic lenses of 23.3 mm aperture and 110 mm focal length, or an anastigmat by Goerz of 20 mm aperture and 180 mm focus could be used. The camera was supplied with a parallactic mounting and driven by a clock-work, but it had no slow-motions for accurately following the movement of the comet, and the arrangements for focusing the camera were not adequate. The views of Halley's comet obtained in Germany in the first months of the year were so unpromising that there seemed to be no reason for equipping the expedition more extensively in this respect, and the principal weight was attached to other problems.

The photographs of the comet obtained by the expedition were a surprise to anyone who had experienced the disappointment as to the appearance of the comet in Germany; but they are indeed not comparable with the plates obtained particularly at the American observatories. However, we were able to derive from them results which appeared to us of interest, in that we could conclude as to the conditions of brightness in the tail of the comet from the blackening of the images on the plate, the necessary data for the photometric comparison having been provided for. Two exposures of the comet were always made simultaneously on the same plate with the two precisely similar Zeiss aplanats, the objective of one having been stopped down to diminish the brightness by 0.4 magnitude.[2] For the plates taken with the Goerz anastigmat,

[1] Translated from authors' proof of a paper read before the Kgl. Gesellschaft der Wissenschaften zu Göttingen, 1911.

[2] In consequence of an oversight this reduction was used, instead of that of 1 magnitude, which was intended.

comparison scales were made with a tube photometer, constructed by Professor Eberhard, on plates of the same emulsion which were developed at the same time with the plates of the comet.

In view of the lack of sharpness of the plates, it would have been of no purpose to derive from them the brightness at every point of the tail. It is also questionable, in view of the complicated structure of the tail, whether much could have been gathered from such a measurement even in case of the best plates, but the following question of a more summary nature seemed more promising.

The brightness of the tail diminishes continuously from the head of the comet outward, aside from irregularities. This is necessarily in part due to the fact that the density decreases in the tail. The decrease of density occurs (1) because the material streaming out from the head is spread out over a constantly increasing cross-section, and (2) because of the simultaneous increase of the velocity with which the particles of the tail passed through this cross-section in consequence of the repulsive force exerted by the sun. Now the question is whether these two circumstances are sufficient to explain completely the diminution of brightness of the tail in a quantitative manner, whence would then follow that the luminosity per unit mass of the material of the tail would remain undiminished during its travels in space; or, on the other hand, whether the decrease in the brightness of the tail is much more rapid than that of the density, which we should a priori regard as more probable for a tail showing essentially a gaseous spectrum, hence a so-called self-luminous tail.

As is evident from the enormously low density of the tail (see below), we observe at each point of the tail the sum of the intensities of the particles lying in the cone of vision, and there is no thought of one particle covering or absorbing the radiation of others in an appreciable degree. We also point out at once that the change of the distance from the sun and hence of the illumination by the sun makes little difference for the relatively short portions of the tail available for measurement.

For settling the above question it suffices to determine "sectional intensities" of the tail, to make a set of photometric

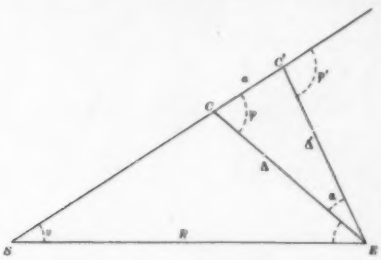
measurements of the tail on a line perpendicular to the axis of the tail, and to find the sum or the integral of the intensities found along each such line. The lateral expansion of the tail no longer affects such sectional intensities, and they are dependent only on the velocity of the streaming in the tail, as to which we have sufficient data from other sources.

Table I contains a number of these sectional intensities derived from the photographs of the expedition to Teneriffe. The unit is arbitrary, varying from day to day. The distance, a, of each cross-section from the head as measured on the plate, is given in degrees. For the determination of these sectional intensities plates that are not very sharp are rather of advantage, inasmuch as they themselves integrate the small irregularities of the structure of the tail.

The following corrections have already been applied to these figures: The intensity of the sky background was determined from the blackening of the plate away from the comet for each section, and was substracted from the total measured intensity. The extinction in the earth's atmosphere for the wave-length 430 $\mu\mu$ was derived from the observations of the sun made on the same day at the same station by Messrs. Müller and Kron. On May 14 only was the weather uncertain and the extinction abnormally large, so that it is easily possible that the comet's head was still more weakened at its low altitude on this day by the mist than would follow from the extinction determined for that day. This would make the intensity for the particles of the tail at a higher altitude still too large in spite of the correction for extinction, and the results of May 14 have accordingly been subsequently given only one-half weight.

It was further to be noticed that the distance Δ' of the different portions of the tail from the earth is not the same, and that the cross-section perpendicular to the apparent axis of the tail as seen from the earth in fact forms a varying angle with the axis of the tail. A change in the distance Δ' causes no change in the surface brightness, but the cross-section of the tail varies inversely proportional to Δ'. Hence all the sectional intensities are to be multiplied with Δ' to reduce them to the same distance. Further, if the angle of

the axis of the tail with the line of sight is ψ', the cross-section will be increased as compared with the normal view in proportion to $1/\sin\psi'$, if we regard the tail as cylindrical for a short distance. Hence the sectional intensities are to be multiplied by $\sin\psi'$, in order to reduce them to sections perpendicular to the axis of the tail. The curvature of the tail amounts to only a few degrees in the case of the distances in question here, so that the axis of the



tail CC′ may be regarded as coinciding with the prolongation of the radius vector SC from the sun to the comet. But we derive from the triangle SEC′ (sun-earth-particle)

$$\Delta' \sin\psi' = R \sin v.$$

R and v, and hence also the quantity $\Delta' \sin\psi'$, are constant for each plate as we proceed along the tail. Hence it follows that the two effects just considered do not together alter the relations of the sectional intensities which mutually counterbalance in so far as they here come into consideration.

Finally, we investigated (by direct plates of a surface of constant brightness) how the surface brightness of the image depends on the distance from the axis for the objectives employed, and we applied for this a small correction.

The numbers in Table I accordingly give the actual relations of the sectional intensities at the time of each exposure.

We have now to investigate how far the sectional intensities contained in Table I are affected by the increasing velocity of the stream of the tail. If I is the sectional intensity and V the average velocity of the stream through the cross-section, then we must have

$$I \cdot V = \text{a constant}, \qquad (1)$$

provided that the stream is stationary and that each particle of the tail retains its luminosity unaltered.[1]

TABLE I

Date	May 1	May 3	May 7	May 8	May 9	May 9	May 10	May 12	May 14
Objective	Zeiss	Zeiss	Zeiss	Zeiss	Zeiss	Goerz	Goerz	Zeiss	Zeiss
Distance from Head a	Intensity I of Cross-Section								
0°.26	....	1.57	....	....	....	....	....	....	....
0.42	1.06	....	....	....	....	....	....	....	....
0.52	....	1.11	2.33	2.78	2.52	2.89	2.65	2.23	(2.82)
0.68	1.27	....	....	....	....	....	....	....	....
0.78	....	0.76	1.87	1.71	2.00	2.30	2.05	1.69	(2.56)
1.04	....	0.72	1.38	1.29	1.42	1.93	1.64	1.47	....
1.20	0.75	....	....	....	....	....	....	....	....
1.30	....	0.70	1.30	1.30	1.32	1.64	1.41	1.31	(2.23)
1.56	....	....	1.24	1.06	1.04	1.38	1.25	1.21	....
1.72	0.61	....	....	....	....	....	....	....	....
1.82	....	0.58	....	....	....	....	....	....	....
2.08	....	....	1.01	1.07	0.81	0.97	1.03	1.02	(2.35)
2.34	....	0.44	....	....	....	....	....	....	....
3.13	....	....	1.00	0.88	0.69	0.48	0.77	0.77	(2.05)
4.17	....	....	0.80	0.70	0.53	0.30	0.61	0.74	(2.09)
5.21	....	....	0.74	0.55	0.41	0.26	0.50	0.64	(1.60)
6.25	....	....	0.74	0.45	0.32	0.28	0.45	0.56	(1.70)
7.29	....	....	0.68	....	....	....	....	....	(2.01)
8.34	....	....	....	....	....	....	....	0.49	....

As to the velocity V, we may assume for the small portion of the tail here in consideration that the repulsive force of the sun is constant and operates in an invariable direction, so that the motion of the particles of the tail relatively to the head is simply a motion of free falling, but solely in the direction away from the sun. The initial velocity being assumed zero, we therefore have for it the formula,

$$V = \text{constant} \cdot \sqrt{a}.$$

In *Publications of the Astronomical Society of the Pacific* (**22**, 125, 1910) H. D. Curtis has communicated the following summary of the observed velocities V of the separate knots in the tail. The values designated by him as uncertain have been omitted.

[1] This equation is also valid if several streams of different velocities are superposed, provided that the velocity in all cases increases in the same relation with increasing distance from the head.

TABLE II

Date	a	V km/sec	$\frac{V}{\sqrt{a}}$
May 2 to 3	0	0.9	...
May 13 to 14	0.0032	4.9	87
May 27 to 28	0.0042	12.8	198
May 25 to 26	0.0100	19.3	193
June 2 to 3	0.0147	31.8	263
May 28 to 29	0.0186	36.8	270
May 31 to June 1	0.0191	33.3	241
June 6	0.0237	43.8	285
May 26 to 27	0.0270	38.4	234
June 6 to 7	0.0438	70.1	331
May 30 to 31	0.0712	72.4	271
June 7 to 8	0.0905	91.2	303

If we form the quotient $\frac{V}{\sqrt{a}}$, we see that for distances from the head greater than 0.01 the values are nearly constant; also that we may infer from the observations at most a tendency to a more rapid increase of the velocity with the distance from the head as compared with the parabolic formula.

If we assume the parabolic formula, and accordingly form the products $I \cdot 1\sqrt{a}$, then we obtain the numbers in Table III, in which for clearness the value of the product for a distance from the head of 0°.52 (1 mm on the plates taken with the Zeiss aplanat) has been placed equal to unity. The last column contains the geometrical mean from the results of the separate plates, in which the few values of May 1 have been omitted and those of May 14 have been given only one-half weight.

Contrary to our expectation, it follows from these figures that *the decrease in brightness of the comet's tail is for the most part explained by the falling off in density.* We cannot insist on a precise constancy of the numbers, for the actual relations in the comet's tail are too complicated and too little explained: first of all the streaming is not stationary, as was assumed in deriving equation (1). With increasing activity of the comet, it is presumable that the portion of the tail nearest to the head would become brighter; and with decreasing activity, that the end of the tail would retain a relatively greater brightness. It is also difficult to estimate the possible magnitude of the systematic errors of our measurement.

TABLE III

Date.....	May 1	May 3	May 7	May 8	May 9	May 9	May 10	May 12	May 14	May 13-14
Objective..	Zeiss	Zeiss	Zeiss	Zeiss	Zeiss	Goerz	Goerz	Zeiss	Zeiss	
Distance from Head a	$I\sqrt{a}$									Geometric Mean
0°.26.....	....	0.63	....	....	....	....	....	....	....	....
0.42.....	0.84	....	....	....	....	....	....	....	....	....
0.52.....	....	1.00	1.00	1.00	1.00	1.00	1.00	1.00	(1.00)	1.00
0.68.....	1.29	....	....	....	....	....	....	....	....	....
0.78.....	....	0.83	0.98	0.75	0.97	0.97	0.95	0.93	(1.11)	0.92
1.04.....	....	0.91	0.84	0.65	0.79	0.94	0.88	0.93	....	0.84
1.20.....	1.20	....	....	....	....	....	....	....	....	....
1.30.....	....	0.99	0.88	0.74	0.83	0.90	0.84	0.93	(1.25)	0.89
1.56.....	....	....	0.92	0.66	0.71	0.83	0.82	0.93	....	0.81
1.72.....	1.16	....	....	....	....	....	....	....	....	....
1.82.....	....	0.98	....	....	....	....	....	....	....	....
2.08.....	....	....	0.95	0.77	0.64	0.67	0.78	0.91	(1.66)	0.84
2.34.....	....	0.85	....	....	....	....	....	....	....	....
3.13.....	....	....	1.05	0.78	0.67	0.41	0.71	0.84	(1.77)	0.77
4.17.....	....	....	0.97	0.71	0.60	0.30	0.65	0.93	(2.09)	0.71
5.21.....	....	....	1.10	0.56	0.44	0.34	0.59	0.87	(2.08)	0.66
6.25.....	....	....	1.00	0.63	0.51	0.29	0.60	0.91	(1.79)	0.66
7.29.....	....	....	1.09	....	....	....	....	....	(2.66)	....
8.34.....	....	....	....	....	....	....	....	0.88	....	....

The constancy of the numbers would moreover be better in the mean if we should give a higher value to the increase of velocity than in the case of a freely falling body, according to the data of Curtis. But even if we select the series decreasing most rapidly, that of May 9, we should still derive from it that the luminosity of the particles of the comet had only decreased to one-third of its value in a period of three days: for this length of time is required according to Curtis' velocities for the particle to proceed to a distance of about 6° from the head of the comet. It appears hardly probable that such a duration of the luminosity could be disputed from the energy of the matter in the tail. But, contrariwise, *our result is immediately explained if we assume that in the tails of comets we are concerned with fluorescent or resonance radiation excited by solar radiation*. Wright (*Lick Observatory Bulletin, No.* 174) very probably observed the resonance radiation of sodium in the spectrum of the tail of Comet 1910 *a*. We should have to assume an analogous origin for the band spectum of *CO* and *CH* in the tail of Halley's comet. The intensity of fluorescence per unit of mass

would further not be constant along the tail, but would decrease with the square of the distance from the sun. This gives on the average for the above days of observation a decrease of about 15 per cent for an apparent distance of 6° from the head. This therefore makes little difference qualitatively, but conforms in sense with the observed values.

In what precedes we have investigated only the relative brightness of the different parts of the tail, but the photographs of the expedition to Teneriffe also enable us to derive the absolute brightness in consequence of the fact that on most of the pictures the stars have, in consequence of inaccurate focus, disks of sufficient size so that the diameters (0.1 to 0.3 mm) can be measured, as well as their amount of blackening. Table IV, which follows, gives for each date the comparison star used, and its magnitude according

TABLE IV

Date	Comparison Star	Magnitude	i	a	I	E_0
		m				
May 1.....	10 *Piscium*	4.38	$0.84 \cdot 10^{-11}$	1°0	$1.8 \cdot 10^{-11}$	$2.2 \cdot 10^{10}$
May 3.....	"	4.38	0.70	1.2	1.4	1.7
May 7.....	35 *Piscium*	5.93	1.14	1.5	1.8	1.9
May 8.....	"	5.93	0.89	1.6	1.1	1.1
May 9.....	"	5.93	1.43	1.7	1.7	1.7
May 12....	*x Piscium*	6.28	1.41	2.0	2.1	1.5
May 14....	"	6.28	0.75	2.2	2.3	1.2

to the *Göttingen Actinometry* reduced to the Harvard system. The mean surface-brightness of the sun was chosen as the unit of surface-brightness, and the value −25.8 mag. was adopted as the photographic stellar magnitude of the sun on the Harvard system, as given by O. Birck in his Göttingen dissertation in 1908. From this was derived the surface-brightness of the comet's tail at the brightest point of the cross-section at a distance of 0°52 from the head; the results are contained in the fourth column of Table IV. The sectional intensities can of course be reduced to the surface-brightness of the sun by applying the same factors. In the column headed a will be found the apparent distance from the comet's head of a point on the radius vector of the comet at an actual distance from it of 0.015 astronomical units. Under the heading I will be found

the sectional intensities interpolated from Table I for this distance from the head, and referred to the surface-brightness of the sun.

As soon as we attribute the luminosity of the tail to the reflection or resonance of solar radiation, the absolute surface-brightness of the tail gives us a datum for the density of the material of the tail; and the absolute sectional intensities can be used in connection with the determinations of velocity, in order to get some idea as to the amount of matter given off by the comet in forming its tail. Let us at first imagine that the tail consists solely of spherical particles of radius ρ which uniformly diffuse the sunlight in all directions[1] without loss by absorption. Let n be the number of particles to the unit of surface, as seen from the earth; and A be the solar radius, and let the distance of the portion of space from the sun be r, as above. Then the surface-brightness as compared with the surface-brightness of the sun is

$$i = n\pi\rho^2 \frac{A^2}{2r^2}. \tag{2}$$

If q is the co-ordinate perpendicular to the radius vector and to the line of sight, the sectional intensity will be

$$I = \int i dq = \pi\rho^2 \frac{A^2}{2r^2} \int n dq. \tag{3}$$

If we return now to only the actual particles which form the comet's tail and to which we may give a radius ρ_0 and a specific weight s, then we may still ascribe to each particle a certain "effective" radius ρ such that the above formulae will be satisfied. The magnitude $\frac{\rho^2}{\rho_0^2}$ will then represent a sort of albedo. The total mass of the comet's tail which will be projected on the unit of surface as seen from the earth will be

$$\tfrac{4}{3}\pi\rho_0^3 sn = \tfrac{8}{3}\rho_0 \left(\frac{\rho_0}{\rho}\right)^2 s \frac{ir^2}{A^2}.$$

If we imagine the tail of the comet so compressed along the line of sight that it assumes throughout the maximum density prevailing in its central portion, and if we call the resulting thickness

[1] Forward as well as backward, as particles do whose diameters are less than the wave-length of the light.

of the tail in the direction of the line of sight, p, then we obtain for this maximum density

$$\delta = \tfrac{4}{3}\frac{\pi\rho_0^3 sn}{p} = \tfrac{8}{3}\frac{\rho_0}{p}\left(\frac{\rho_0}{\rho}\right)^2 s\frac{ir^2}{A^2}. \qquad (4)$$

The total mass which passes through the cross-section of the tail in a unit of time, or the "productiveness," becomes

$$E = \frac{4\pi}{3}\rho_0^3 s \cdot v \int ndq = \tfrac{8}{3}\rho_0\left(\frac{\rho_0}{\rho}\right)^2 sv\frac{Ir^2}{A^2}.$$

The unit of length employed above in computing the sectional intensity was 0.2 mm on the plates taken with the Zeiss aplanat, corresponding to 6′.2. The actual magnitude of this unit of length is $\Delta \sin 6'.2$, Δ being the distance of the comet. The above quantities I are therefore still to be multiplied by this. If we finally apply the factor $\sin\psi$, which as stated reduces to normal presentation, then the complete formula for computing the productiveness from the sectional intensity I of the table is

$$E = \rho_0\left(\frac{\rho_0}{\rho}\right)^2 sE_0, \quad E_0 = Iv\frac{r^2}{A^2}\Delta \sin\psi \cdot \sin 6'.2. \qquad (5)$$

The figures of Curtis for the velocity of streaming at a distance of 0.015 astronomical unit from the head come out about $v = 32$ km per second. With these data the values E_0 of Table IV were computed in units of the C.G.S. system. On the average we therefore have, approximately,

$$E = 1.5 \times 10^{10}\rho_0\left(\frac{\rho_0}{\rho}\right)^2 s.$$

If we assume that the section of the tail in space is of approximately cylindrical form, then we should derive from a side view of the tail at 0°.5 distance from the head a thickness of the tail p (in the above explained sense) of about 300,000 km (3×10^{10}cm). Hence with the use of the average value $i = 1.0 \times 10^{-11}$ from Table IV, it follows that

$$\delta = 2 \times 10^{-17}\rho_0\left(\frac{\rho_0}{\rho}\right)^2 s.$$

We shall now follow out two assumptions as to the nature of the particles:

1. In accordance with the theory of Arrhenius as to light-pressure, let the particles be of the order of magnitude $\rho_0 = 10^{-4}$ cm,

the specific weight being one. The effective radius for such particles will be of the same order as the actual radii, and we will here simply place $\rho=\rho_0$. We here overlook the fact that such particles should chiefly reflect the solar spectrum instead of giving a band spectrum, and we ought to take into account in our computation for them only the fraction of the light of the tail which consists of reflected sunlight.

2. We regard the particles as fluorescing molecules whose radius, ρ_0, is then of the order 10^{-8} cm. It is not improbable that the effective radius of such a molecule is similarly of the same order of size as the actual radius,[1] so that we again place $\rho=\rho_0$ and take 20 as the molecular weight. In the two cases we obtain:

HYPOTHESIS I

$E=1500$ kilograms per second,
$\delta=2\times10^{-21}$;

HYPOTHESIS II

$E=150$ grams per second,
$\delta=4\times10^{-24}$.

The free path of the molecules at the last density would be of the order of 10^{19} cm, corresponding to the distance of the nearest fixed stars, so that collisions would no longer occur.

If we expose the earth for a whole day to a current of density 10^{-22} having a velocity of 100 km per second (these figures must roughly hold good according to Curtis' velocities and according to Hypothesis I for the transit of the earth through the comet's tail), then on each square centimeter of the earth's surface there will fall a mass of 10^{-10} grams, carrying an energy of 1/10,000 gram-calories. The total mass caught by the earth amounts then to 250,000 kilograms. Since these figures may be regarded as upper limits, we see how little material action from the transit by the comet's tail was to be expected.

POTSDAM
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[1] This holds good at least if the fluorescence is due to the fact that a few electrons in the molecule damped only by radiation swing about their position of rest with frequencies of the order of those of light.

A DETERMINATION OF THE MELTING-POINTS OF TANTALUM AND TUNGSTEN

By WILLIAM E. FORSYTHE

INTRODUCTION

In measuring temperatures much above 1550° C., the limit so far attained with the constant-volume nitrogen thermometer, the best available method is to determine the temperature in terms of the radiation from a standard source. The standard source that is taken is Kirchhoff's "black body," and the best way of expressing the temperature, at least tentatively, is in terms of a temperature-scale based on either the Stefan-Boltzmann or Wien law of radiation. The usual form of the Stefan-Boltzmann law is

$$E=A\ (T_1T_1^4-T_2T_2^4),$$

where E is the total radiation sent out from the radiator at temperature T_1, to the receiver at temperature T_2, A being a constant. The use of this law requires a measurement of the total radiation falling upon the receiver.

Wien's equation is

$$E_\lambda=c_1 \cdot \lambda^{-5} \cdot e^{\frac{-c_2}{\lambda T}},$$

where E_λ is the energy sent out from the radiator corresponding to the wave-length λ and the temperature T, c_1 and c_2 being constants. The use of this equation requires a measurement of the energy corresponding to a particular wave-length.

The temperature-scales as defined by these two relations agree with the nitrogen thermometer up to 1550° C. and have been shown by Lummer and Pringsheim[1] to agree with each other up to about 2000° C. and by Gillett[2] to be in approximate agreement up to about 2500° C.

The temperature of a non-black radiator obtained by measuring the energy corresponding to a particular wave-length is called the

[1] *Verhandlungen der deutschen physikalischen Gesellschaft*, **13**, 36, 1911.

[2] *Journal of Physical Chemistry*, **15**, 213, 1911.

"black-body" temperature and may be defined as the temperature at which a black body would send out the same amount of radiation of this wave-length. The black-body temperature of any substance will depend upon the condition of the surface, the departure from the true temperature being greater the higher the reflecting power. Since the black-body temperature in general depends upon the wave-length considered, in giving this temperature the wave-length should be specified.

Thus, to determine the true temperature of the melting-point of a metal with an optical pyrometer, either the metal may be melted in a black-body furnace and the temperature taken by observations of the furnace through a small opening, or the black-body temperature of the melting-point may be found by taking observations upon the melting metal with the optical pyrometer, and the difference between the black-body and the true temperature determined by some other means.

The present work gives the results of the direct measurement of the melting-point of tungsten in a black-body furnace and of tantalum and tungsten by a filament method, together with measurements of the black-body melting-point of both tantalum and tungsten by several methods. These black-body melting-points have been reduced to true melting-points by a method to be described later.

ERRORS

An error may be introduced in measuring the temperature of a black-body furnace due to the furnace not being in condition to give black-body radiation, which would make all temperatures as measured from it too low. The furnace will not give out black-body radiation if its walls are not uniformly heated or if there are too many or too large openings in the walls. A simple qualitative test of blackness is given by the degree of invisibility of objects of different radiating powers placed within the furnace. The condition of blackness of the furnace should be further tested by observations at some known temperature.

A second source of error may be introduced by faulty calibration of the pyrometer. This may be due to an error of setting the comparison lamp for a balance with the standard black-body

furnace; but if several readings are taken, and care is exercised always to use the same part of the filament of the comparison lamp,

TABLE I

COMPARISON OF PYROMETERS

Source	Spectral	With Glass
Small furnace	1062° C.	1062° C.
Small furnace	1218	1218
Small furnace	1386	1387
Nernst glower	1861	1858
Nernst glower	1995	2000
Nernst glower	2080	2078
Nernst glower	2080	2085
Nernst glower	2174	2172
Graphite tube furnace	1990	1990
Graphite tube furnace	2370	2380
Graphite tube furnace	2480	2490

this error may be very much reduced. Also the temperature of the standard black body used in calibrating the pyrometer may not be known accurately—an error of one degree at 1500° C. leading to an error of 3° C. at 3000° C.

In measuring temperature with an optical pyrometer an error may be introduced in determining the wave-length used with the colored glass, as the position of the maximum effective transmission of the glass may depend upon the temperature of the source, that is, on the distribution of energy in the source. To overcome this difficulty Professor Mendenhall[1] designed a direct-vision prism spectroscopic pyrometer by means of which observations can be made with light having a spectral range of not more than 25 Ångström units. Setting this pyrometer for $\lambda = 0.658\ \mu$, which was the wave-length corresponding to the position of the maximum transmission of the colored glass used on the other pyrometer, the spectral pyrometer was calibrated and the two pyrometers compared. In Table I are compared the results for the different temperatures and different sources taken with the spectroscopic and regular pyrometer, which show that within the limits of setting the two agree.

In any determination of a melting-point an error is always liable to be introduced due to an uncertainty as to the exact time of

[1] *Physical Review*, **33**, 74, 1911.

melting; furthermore, some chemical reaction may take place in the furnace between the metal and some gas or vapor that may be present, thus raising or lowering the melting-point. The method taken to guard against the above errors will be pointed out farther on.

In comparing high-temperature measurements there is much confusion due to the use of three different values for the melting-point of platinum. In Germany two temperature-scales are in use, the one giving the melting-point of platinum as 1788° C., the other placing this temperature at 1745° C. In this country most of the work is referred to the melting-point of palladium at about 1549° C. and that of platinum at about 1755° C., and in this paper we shall accept these, which are the values of Day and Sosman.[1] The constant c_2 of Wien's equation will be taken as 14,500, while the wave-length used with the optical pyrometer was 0.658 μ.

PREVIOUS DETERMINATIONS

In recent years there have been several determinations of the melting-points of tantalum and tungsten by indirect methods requiring extrapolation of several hundred degrees to reach the true melting-point. The first attempt to measure these high temperatures was in 1906 by Waidner and Burgess,[2] who found a value for the black-body melting-point of tantalum and tungsten by obtaining an equation between the current and the black-body temperature of a filament in a commercial lamp and then noting the current required to melt the filament. As this equation was obtained from measurements made below 1900° C., an extrapolation of about 1000° C. was required to reach their values for the black-body melting-point. To obtain the true temperature they assumed for tantalum and tungsten a relation between the black-body temperature observed with red light and the black-body temperature observed with blue light, a relation which had been found to hold very well for platinum. Their final value was 3080° C. for the melting-point of tungsten, and 2910° C. for the melting-point of tantalum. The method they used to obtain the relation of the

[1] *American Journal of Science* (4), **29**, 93, 1910.

[2] *Journal de Physique* (4), **6**, 830, 1910.

black-body temperature to the true temperature may give rise to a considerable error, and moreover von Wartenberg[1] has shown by direct observation that the black-body melting-point of unpolished tungsten is about 100° C. lower than the value they obtained.

In 1910 von Pirani[2] found for both tantalum and tungsten a relation between the true temperature and the black-body temperature by measuring the true temperature with a thermocouple between twisted filaments of the metal, and the black-body temperature with a Wanner pyrometer sighted at these twisted filaments. Whether the thermocouple would give the true temperature, or a temperature higher or lower than that of the surface, would depend upon how closely the wires were twisted together, while without doubt the black-body temperature as measured would be too high, due to the fact that the openings between the twisted filaments would more or less approach a black body in their radiation. These measurements on the true temperature and the black-body temperature were made below 1700° C., and to extrapolate to the melting-point it was assumed that the energy used in the filament varied as the mth power of the absolute temperature, that is:

$$E = e \cdot i = AT^m, \text{ or}$$
$$\log ei = A^1 + m \cdot \log T,$$

where e is the applied voltage and i the current through the filament. This equation, which is linear in $\log ei$ and $\log T$ was found to hold over a small range of temperature, about 500° C., and was then assumed to hold up to the melting-point, which was 1300° higher. After applying a correction, Pirani's final result was 3250° C. for tungsten and 3000° C. for tantalum, using the scale giving the melting-point of platinum as 1745° C. This correction, of about 200°, added to the results as obtained by extrapolation, was due for the most part to two causes: the first being the conclusion that his temperatures as measured around 1600° C. were too low and the second the fact that in the equation $\log ei = A^1 + m \cdot \log T$, m cannot remain constant but must decrease with increasing temperature. A similar extrapolation gave for Waidner and Burgess (*loc. cit.*) a result for the black-body temperature of the

[1] *Berichte der deutschen chemischen Gesellschaft*, **40**, 3287, 1907.

[2] *Verhandlungen der deutschen physikalischen Gesellschaft*, **12**, 301, 1910.

melting-point that seems to be about 100° too high, so that a like error may be introduced here. As Pirani gives no values for the black-body melting-point, a direct comparison cannot be made.

Later,[1] with somewhat similar extrapolations, Pirani found the black-body melting-point of tantalum and also measured this melting-point with a pyrometer. From this black-body melting-point and the reflecting power of tantalum the true melting-point was found by the same method as that used by von Wartenberg, which is described below. This result seems to be free from some of the objections stated above, as no corrections due to errors in the method were made.

Wartenberg (*loc. cit.*) in 1910 obtained the black-body melting-point of tungsten by melting this metal in a vacuum tube, in which a small piece of tungsten was mounted as an anode; the discharge from a Wehnelt cathode sent down upon it raised the temperature to the melting-point. Later he[2] measured the reflecting power of tungsten and was thus able to apply the equation used by Holborn and Henning to compute the true temperature. This equation, derived directly from Wien's radiation equation, is

$$1/S - 1/T = c_2/\lambda \cdot \log A/\log e,$$

where T is the true temperature, S the black-body temperature, c_2 the constant of Wien's equation, λ the wave-length used, A the absorbing power, that is $(1-R)$, where R is the reflecting power. Wartenberg tested this relation for several different metals and found that it held within his limits of error up to about 2400° C. By applying this equation to his values for the black-body melting-point of tungsten he obtained about 2930° C. for the true melting-point.

The only attempt to measure the melting-point of tungsten and tantalum by a direct method was by Ruff[3] in 1910, using a vacuum furnace with a heater tube of carbon. In this furnace the heater tube had in it longitudinal slots that served the double purpose of allowing observations to be made upon the temperature inside and

[1] *Verhandlungen der deutschen physikalischen Gesellschaft*, **13**, 540, 1911.

[2] *Ibid.*, **12**, 105, 1910.

[3] *Berichte der deutschen chemischen Gesellschaft*, **43**, 1564, 1910.

also of increasing the resistance. The metals studied were placed in the furnace in the form of Seger cones and the temperature taken when the cones toppled over. Ruff obtained for the melting-point of tungsten by this method about 2650° C. The work of Wartenberg (*loc. cit.*) indicates that this is far too low, as it is 100° C. lower than the black-body melting-point.

DIRECT DETERMINATION

The vacuum furnace used in this work was designed in this laboratory by Mr. Steve and Mr. Barnes along the same general lines as the furnace of Arsem[1] of the General Electric Company and that of King[2] of the Mount Wilson Solar Observatory.

In this furnace, as shown in Fig. 1, the terminals, heater tube, etc., were all inclosed in an air-tight, water-cooled jacket made of iron pipe. The jacket was mounted vertically, with all the working parts of the furnace fastened to the top plate so that they could be easily removed. In order to be able to observe the inside of the furnace there was a window in each end 6 cm in diameter covered with plate glass 0.7 cm thick, held in place between two rubber gaskets. The terminals consisted of copper tubes 1.3 cm in external diameter and 1 cm in internal diameter, each terminal being made of two of these tubes, for inflow and outflow of cooling-water, which were connected at the bottom with a hollow brass casting, to which was fastened the graphite clamps for holding the heater tube. The graphite clamp just fitted the ends of the heater tubes of Acheson graphite which were obtained from the Acheson Graphite Company, turned to the size and shape desired. The heater tubes were 30 cm long, 1.4 cm in internal diameter, and 1.9 cm in external diameter, excepting about 2.5 cm at each end which were 2.5 cm in diameter. This made the thickness of the hot part of the tube wall about 0.3 cm. No allowance was made for the expansion of the heater tube other than slipping in the graphite clamps. To insulate the heater tube from losses of heat to the sides of the jacket it was surrounded by two carbon tubes, as shown in the figure, it being found that when the inner insulating tube was removed the

[1] *Transactions of American Electrochemical Society*, **9**, 153, 1906.

[2] *Astrophysical Journal*, **28**, 300, 1908.

maximum temperature obtainable with a given applied voltage was about 400° C. lower than when it was in place.

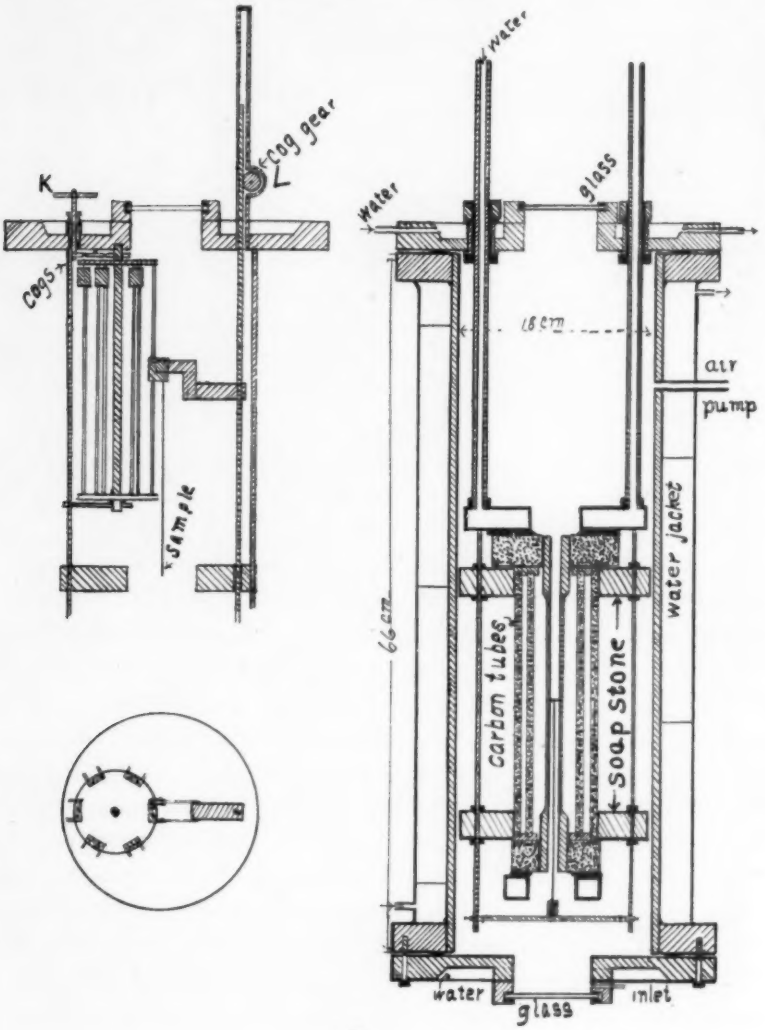

FIG. 1

The heating current was taken from a transformer, the current through the primary being regulated by two electrolytic rheostats of copper sulphate solution in earthenware tanks. These two

rheostats were connected in parallel so that a small variation of current could be obtained and the temperature easily controlled. Most of the high-temperature work was done with the transformer set so as to give 30 volts from the secondary, and with this arrangement and all the primary resistance in, the primary would draw about 150 amperes, which gave about 550 amperes through the heater tube. Table II gives the current through the heater tube, the voltage across the copper leads, and the resulting temperature of the furnace. If the temperature of the furnace was not raised above 2950° C. a heater tube would stand two or three runs; i raised much higher would stand but one run, and at a temperature higher than 3000° C. the tube would last but a few minutes.[1]

To obtain the vacuum in the furnace an electrically driven Fluess air-pump was used giving a vacuum of a few tenths of a millimeter pressure, which we were able to hold in the furnace for several days. However, when the furnace was run at the high temperatures, the pressure was hardly ever less than a few millimeters.

TABLE II

Current through Heater Tube	Voltage across Copper Leads	Temperature of Furnace
550 amperes	10 volts	1500° C.
790	13	1780
1000	16	2110
1100	18.3	2425
1175	20	2650
1240	21.5	2900
1400	25	3100

A disk of graphite about 0.1 cm thick was placed inside the heater tube somewhat nearer the lower end of the tube and held in place by a graphite rod 0.6 cm in diameter at the lower end and 0.2 cm in diameter at the top, this rod being insulated from the remainder of the furnace to prevent the formation of an arc at the center of the heater tube.

That the necessary conditions for black-body radiation were satisfied was shown by a careful determination of the melting-point of platinum with a calibra ted optical pyrometer. The results

[1] Watts and Mendenhall, *Physical Review*, **33**, 65, 1911.

indicated a very satisfactory condition of blackness, since the value obtained for the melting-point agreed well with the best accepted values. Again, after the furnace had been run on steady current for a few minutes no difference could be detected between the temperature of the disk and the walls of the heater tube; furthermore, small pieces of magnesium oxide, small pieces of tungsten filaments, and a tungsten cone could not be seen when in the furnace unless they projected over an opening between the disk and the walls of the heater tube.

The specimens to be melted were carried by an arrangement shown in Fig. 1, the specimens being in the form of hairpin filaments 0.014 mm in diameter, the free ends being fastened to the two arms of the carriage. When a specimen was wanted in the furnace it was turned in place by the handle K and then pushed down in the furnace by the handle L. The specimen s completed an electric circuit so connected to a relay that a bell would ring when the specimens melted and broke the circuit.

The temperature of the heater tube was measured with an optical pyrometer sighted through the plate-glass window on the disk in the heater tube. This made necessary a correction for the light absorbed and reflected by the plate glass which was easily determined by measuring first the temperature of some steady source with the pyrometer and then the apparent temperature of the same source when sighted through the plate glass. For the first case, from Wien's equation,

$$E_1 = c_1 \cdot \lambda^{-5} \cdot e^{\frac{-c_2}{\lambda T_1}},$$

where E_2 is the energy corresponding to the black-body temperature T_1. For the second case, through the plate glass,

$$E_2 = c_1 \cdot \lambda^{-5} \cdot e^{\frac{-c_2}{\lambda T_2}},$$

where E_1 is the amount of light passing through the plate glass from the source at the black-body temperature T_1, T_2 being the (apparent) black-body temperature corresponding to E_2. Taking the logarithms of these two equations and substracting the first from the second,

$$\log [E_2/E_1] = \log K = c\,(1/T_2 - 1/T_1),$$

where K is the fractional part of the light let through by the plate glass and $c=c_2/\lambda(-\log e)$. This equation is linear in $1/T_1$ and $1/T_2$, the graph of $1/T_1$ as a function of $1/T_2$ making an angle of 45° with the axis. Thus a measurement of T_1 and T_2 at but one temperature would give the correction at all temperatures. In practice, however, T_1 and T_2 were measured for several temperatures, using as a source a Nernst glower mounted in a hollow, talc holder. As the glass would get a little fogged at times this correction was determined after each run. For clean glass this correction amounted to about 30° C. at 1600° C. and to about 70° C. at 2800° C.

In measuring the high temperatures use was made of an optical pyrometer of the Holborn-Kurlbaum type.[1] The comparison lamps were 6-volt, 1.3-ampere carbon lamps with pear-shaped bulbs about 2.5 cm in diameter, the horseshoe filament being so mounted that the top of the filament came in the center of the bulb. These lamps were aged for 24 hours on 1.35 amperes before being calibrated. In calibrating these lamps we used the sector method and as the known temperature the melting-point of palladium, this fixed point being determined directly by noting the temperature of melting Kahlbaum palladium. The melting-point of palladium was chosen as the standard temperature because it is about the highest temperature which can be directly observed with a carbon lamp in the pyrometer.

A 1/60 sector was used in the work with platinum, and a 1/180 in the work with tungsten. Starting with 1549° C. (palladium point), the range of the pyrometer with the 1/60 sector was up to 2482° C., with the 1/180 up to 2919° C. The 1/180 sector was 26.8 cm in diameter, which made the opening about 1.7 mm wide at the narrowest point. To determine whether there was an error due to diffraction with this small slit, a sector having ten openings the size of these was made up and points found on the calibration-curve for this sector. No difference was detected in using a sector made up of ten openings and one made with one large opening having the same total area.

Several times the same lamp was calibrated from the gold point

[1] Mendenhall, *Physical Review*, **33**, 74, 1911.

(1063° C.) and from the palladium point, which gave a chance to compare the palladium point with the gold point. As an average of several determinations of the palladium point by extrapolation from the gold point we obtained 1548° C., while an extrapolation down from the palladium point gave 1063° C. for the gold point.

As a test of the furnace and the pyrometer a determination of the melting-point of platinum was made. As it was known that platinum would react with carbon vapor at high temperatures, the platinum was inclosed in small magnesium tubes. The platinum gave, when it broke, all evidence of fusion, the ends being globular and small bright masses of platinum being scattered all along the lower end of the tube. Table III gives the results for the work on platinum in the graphite tube furnace. For the work in the furnace Baker platinum was used and the melting-point of this compared with that of Heraeus by the method of Mendenhall and Ingersoll.[1] The two specimens were melted on a Nernst glower and from their values for the constants of the glower the difference in temperature was calculated. As an average of ten melts the melting-point of Heraeus platinum was found to be six degrees higher than that of the Baker platinum. This gives a final average of 1755° C. for the melting-point of Heraeus platinum, which is in good agreement with the value of 1755° as set by Day and Sosman (*loc. cit.*).

TABLE III

MELTING-POINT OF PLATINUM

1909/12/2	1749° C.
	1744
	1744
	1746
	1744
1910/2/12	1756
	1751
	1749
	1756
Mean	1749° C.

The melting-point of tungsten has been measured in the vacuum furnace described above, indications of a melt being obtained by three different methods. The first was, as outlined above, the use of

[1] *Physical Review*, **25**, 1, 1907.

hairpin filaments, a melt being indicated by the bell ringing when the circuit was broken. The pieces that we were able to get out of the furnace showed every indication of fusion, the ends being tipped with small round bits of the metal. The second method was to bore a small hole in the disk in the center of the heater tube and to lay over this hole a small piece of filament which could be observed to melt and disappear as the temperature was raised. This was undertaken to find out whether direct contact of the tungsten with the carbon made any difference. The results show that the difference, if any, is slight. The third method was to press powdered tungsten into the form of a Seger cone and place this in the heater tube in such a manner that the tip of the cone would project over an opening between the disk and the walls of the tube. The results are given in Table IV.

TABLE IV

MELTING-POINT OF TUNGSTEN IN FURNACE

1911/2/18, Filament	2948° C.
3/1, Filament	2972
3/6, Filament	3010
	3000
3/7, Filament	2948
3/15, Filament	2984
	3000
	3000
3/28, Filament	2917
3/28, Filament on disk	3000
	3000
	2947
3/28, Cone	2937
Mean	2974° C.

MELTING-POINT AS OBSERVED IN A TUNGSTEN WEDGE

8/28	2970° C.

Several attempts were made to melt tantalum in the graphite furnace but with no success. The tantalum was put in the furnace and the temperature run up to between 3000° C. and 3100° C., but when it was taken out it showed no signs of fusion. From the results given below, the melting-point cannot be as high as this. When the tantalum had been in the furnace at this high temperature it did not look at all as it did when put in; indeed, it had more the

appearance of a cinder when broken and observed with a microscope. It would seem that some compound with the carbon vapor was formed which had a higher melting-point than the tantalum.

INDIRECT METHOD

A method of obtaining the relation between the true temperature and the black-body temperature has been described by Professor Mendenhall,[1] tested for platinum, and found to give results that agree very closely with the theoretical and experimental results for this relation. This method consists of making a hollow wedge of a thin sheet of the metal and determining the black-body temperature by observing on the outside of the wedge and the true temperature inside of the wedge.

Such a curve giving the relation between black-body and true temperature for tungsten has been determined by Professor Mendenhall and the writer, while Mr. McCauley assisted with the curve for tantalum. These curves were determined with the metals in the lamp described below. The curve for tantalum was run up to the melting-point and observations made on both the true temperature and the black-body temperature, while the tungsten was run up to a black-body temperature of 2500° C., which corresponds to a true temperature of about 2700° C.

The black-body melting-point of tungsten was found by two methods. In the first place it was determined in a manner similar to that used by Waidner and Burgess,[2] excepting that in this case the lamp filaments were balanced photometrically against the heater tube of the furnace and the temperature taken with the optical pyrometer. Several lamps were worked with, but unfortunately only one lamp melted at the point under observation. This method required a correction for the lens used to project the image of the furnace on the lamp filament and also for the walls of the lamp bulb. This latter correction was determined after the melt in order to correct for any blackening of the bulbs. The ends of the filaments gave indications of a true fusion, being tipped with shiny beads of the metal.

[1] *Astrophysical Journal*, **33**, 91, 1911.

[2] *Bulletin of the Bureau of Standards*, **2**, 319, 1906.

The second method was to mount the metal studied between brass terminals and put these in a water-cooled brass vessel with windows for observing the temperature. The vessel was pumped out with a Gaede pump and calcium heated[1] inside in order to remove the last trace of gas. This vessel was 12 cm in internal diameter and 18 cm high, with windows 2.5 cm in diameter. The lower terminal was connected to the outside with a packed joint which allowed the filament or wedge to be kept straight. After the pump had been run for 24 hours the tantalum would continue for some time at constant temperature, if the current was kept constant; and the drop in voltage across the filament would also remain constant. This was taken to indicate good working condition. The tantalum filaments were rolled from strips obtained from Siemens and Halske, while the tungsten filaments were of two kinds. The first were from a series tungsten lamp from the General Electric Company, and the second were made from ductile tungsten

TABLE V

BLACK-BODY MELTING-POINT OF TUNGSTEN.....$\lambda=0.658$

Filament	2827° C.
Strip	2778
	2793
	2814
	2814
Lamp over furnace	2791
Strip	2791
	2770
	2770
	2791
	2820
	2800
Mean	2797° C.

obtained from the same company. All the filaments used were about 3 cm long, the flat ones from 0.7 to 2.0 mm wide and from 0.05 to 0.3 mm thick, and the filaments from the tungsten lamp were 0.3 mm in diameter. The flat filaments were made a little narrower at the middle so that they would heat more at this point.

[1] Soddy, *Proceedings Royal Society*, A **78**, 429, 1906.

In Table V are given results for tungsten, and for tantalum in Table VI.

Taking the mean of the values for the black-body melting-point of tungsten as given in Table V, and extrapolating the curve showing the relation between the true and black-body temperature to this black-body temperature, we obtained 3030° C. as the true melting-point of tungsten. This is thought to be too high, as the surface of the tungsten used in the work with the wedge changed as a result of being run so long at the high temperatures.

TABLE VI

BLACK-BODY MELTING-POINT OF TANTALUM.....$\lambda = 0.658$

(Figures in parentheses following values indicate weights given)

Strip	2486 (1)
	2517 (2)
	2530 (3)
	2492 (1)
	2515 (2)
	2495 (1)
	2472 ($\frac{1}{2}$)
Mean	2511° C.

MELTING-POINT OF TANTALUM AS OBSERVED FROM WEDGE

	2795° C.
	2786
	2805
	2790 (2)
	2785
	2830
	2805
Mean	2798° C.

DISCUSSION OF RESULTS

The results for the melting-point of tungsten as obtained from the graphite tube furnace are seen to have quite a wide variation. This for the most part is thought to be due to the temperature of the furnace increasing in some cases too rapidly because of varying voltage. In this case the disk would not immediately assume the temperature of the walls of the heater tube. The variation of the observed black-body melting-point of both tantalum and tungsten

may be due to the pyrometer not being sighted at the exact point on the strip at which melting occurred. The mean of the values given for both the black-body temperature and the true temperature are thought to be accurate to about 1 per cent.

TABLE VII

VALUES FOR THE MELTING-POINT OF TUNGSTEN

Observer	Date	Method	Value Accepted for Melting-Point of Pt.	Trials	Value	Average Departure from Mean
Waidner and Burgess	1907	Extrapolation . . .	1753° C.	8	3080° C.	28° C.
Pirani	1910	Extrapolation . . .	1745	. .	3250	. .
Ruff	1907	Carbon-tube furnace	1745–55	. .	2650	33
Wartenberg . .	1910	Black-body melting-point and reflecting power	1745	7	2930	30
This work . . .	1911	Furnace	1755	13	2974	27
This work . . .	1911	Observations on wedge	1755	1	2975	. .
This work . . .	1911	Black-body melting-point and wedge	1755	11	3030	16

VALUES FOR THE MELTING-POINT OF TANTALUM

Waidner and Burgess	1907	Extrapolation . . .	1753	5	2910	. .
Pirani	1910	Extrapolation . . .	1745	. .	3000	. .
Pirani	1911	Black-body melting-point and reflecting power	(*)	14	2700	30
This work . . .	1911	Wedge	1755	8	2798	13

*This value has been reduced to a scale with the melting-point of palladium 1549° C. in place of 1575° C. and $c_1 = 14{,}500$ in place of 14,200.

The results for the true and black-body melting-point of tungsten are seen to be about 100° C. lower than the corresponding temperature as found by Waidner and Burgess (*loc. cit.*). As was pointed out above, their work might have an error of that amount. The value for the melting-point of tungsten as found by v. Pirani (*loc. cit.*) is about 300° C. higher than the results given above. As was shown above, his results are in question due to possible errors. The results for the black-body melting-point of tungsten as given by von Wartenberg (*loc. cit.*) check well with the values given

in this work, the difference in the final values being about that due to the different scales used. This is a satisfactory agreement, as he actually measured the black-body temperature.

As for the results on tantalum, they do not agree with other determinations of this point. Unless the metal changes in some way at the higher temperatures, it is hard to see why there should be this much difference between the several determinations. In Table VII is given a summary of all the work on these two metals, with values found and the scale used.

SUMMARY

The melting-point of tungsten has been found by two methods which are seen to agree within about 1 per cent, the value obtained by measuring the point directly in a black-body furnace being 2974° C., while that obtained from the relation between the true and black-body temperature as measured from the wedge is 3030° C.

The melting-point of tantalum was determined from direct observations upon a melting wedge of the metal, giving a value of 2798° C.

In conclusion I wish to express my thanks to Professor Mendenhall for his many valuable suggestions and assistance in all parts requiring two observers.

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ON THE NEED OF ADJUSTMENT OF THE DATA OF TERRESTRIAL METEOROLOGY AND OF SOLAR RADIATION, AND ON THE BEST VALUE OF THE SOLAR CONSTANT

By FRANK W. VERY

The field of terrestrial meteorology is one of extreme complexity. The several data are so intimately interrelated that changes in one or more of the values will affect the entire series. The *best* value of any one quantity is that which consistently harmonizes the largest number of elements in this complicated interlocking system; and of these related quantities, the one on which many others depend, and the most important astrophysical datum is the solar constant of radiation, now generally recognized as varying slightly, but which nevertheless still maintains a substantial uniformity through the ages which is truly wonderful, and is to be explained only on the supposition that it depends upon some physical constant, such as the boiling-point of the photospheric material or the dissociation temperature of some refractory compound.

The solar constant cannot be directly observed. Its determination, therefore, is open to an unusually wide latitude of interpretation. Such diverse values have been assigned to it that skepticism is natural in regard to even the most plausible results. If the accuracy of the conclusions were at all comparable with the labor which has been expended upon them, we ought to know the value of this fundamental quantity to three or four significant figures, but we are still debating over the first figure.

The reformation of the methods for obtaining the value of the constant introduced by Forbes, given a solid rational basis by Langley, and further improved on the mathematical and observational sides by Crova and Savélief, has apparently been relinquished at the present time in favor of methods which do not differ essentially from those of Pouillet. The elaborate researches of Abbot and Fowle, while purporting to be a continuation and perfection of Langley's methods, are in reality a complete abandonment of the

essential principle which was admirably stated by Langley in his paper "On the Amount of the Atmospheric Absorption,"[1] and which has been repeatedly recognized by many able investigators. The principle is—

> that radiant energy is not a single emanation but the sum of an infinity of diverse ones, each with its own separate rate of absorption. It follows that the coefficient of transmission is truly constant only in the case of the absolutely homogeneous ray, which the ordinary photometer or thermometer cannot in the least discriminate; and which the finest linear thermopile or bolometer can but approximately discern, and hence that the original light of the star or heat of the sun, and the amount absorbed, can at best only be found approximately.[2]

The value of approximately 2 calories which is given in the second volume of the *Annals of the Astrophysical Observatory of the Smithsonian Institution* is to all intents and purposes the same as that of Pouillet; for if the positive corrections which are known to be required for the latter, on account of the defects of the water pyrheliometer, are applied, we shall have a value of the solar constant differing very little from that which is now offered to us as an improvement on Langley's result, but which is quite the reverse. The methods used in the *Annals* have all the characteristics of the original Pouillet model, including the admirable agreement of the separate measures and the small apparent probable error of the final result. The fallacy attending such methods may be recognized when it is known that actual reliable measures of solar radiation may be made within the atmosphere which exceed the supposed value outside the atmosphere, as I shall show in a subsequent paper.

While recognizing the elaborate and painstaking character of the researches contained in the second volume of the *Annals*, I cannot admit that they have fixed even the first figure of the solar constant; but, on the contrary, the necessity of harmonizing all of the data proves conclusively that Langley's original figure is nearer to the truth.

One of the weak points in the argument relates to the absorbent power of aqueous vapor. Acting on the hypothesis that the transmission by the aqueous vapor varies as a power of the experimental

[1] *American Journal of Science* (3), **28**, 163, 1884.

[2] *Op. cit.*, p. 167.

coefficient of transmission in which the exponent of the power is taken equal to the aqueous mass, Abbot and Fowle draw a curve of t to the twenty-sixth power to represent the aqueous transmission of telluric radiation by the air above Mount Wilson. It is true that they cast some doubt upon the justification of the procedure, saying: "It is barely possible that the water-vapor column through which Rubens and Aschkinass observed was so much more dense than that in the atmosphere that its absorption bands were broadened, owing to the great density of the absorbent, so that there was a stronger absorption than an equal amount of vapor would have produced if contained in a long column of less density."[1]

This supposition, which is given as "barely possible," is one which I have already demonstrated. Reference may be made to p. 99 of my work on *Atmospheric Radiation*, published by the United States Weather Bureau in 1900, where it is stated that "with a radiating source at the boiling-point of water, 10 microns of liquid water absorb 156 times as powerfully as the same amount of water distributed through about 100 meters of air." Moreover, the transmission of radiation by aqueous vapor at constant tension does not diminish according to a power of t equal to the depth of the vapor traversed, but the initial absorption of the first part of the vaporous layer immensely exceeds the subsequent absorption. Neither does the vapor have to be of considerable tension in order to be effectual. With a dew-point nearly 30° C. below zero, when only a minute amount of aqueous vapor remains in the air, I have found the great water band Ξ in the solar spectrum exerting almost complete absorption between wave-lengths of 5 μ and 8 μ. Owing to these properties of vaporous absorption, the transmission by the aqueous vapor above Mount Wilson cannot be equated to the twenty-sixth power of the aqueous transmission observed with 75 cm of concentrated steam, but is more nearly identical.

A correction of +18.3 per cent is made in the *Annals* to the Allegheny measures of transmission by the aqueous vapor in 110 meters of air, derived from observations on a distant radiator, and the error is attributed to "stray sunlight (probably reflected from the air)." As these measures were made by a method devised by

[1] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **2**, 169.

me, which I have since perfected, and which, as now modified, gives reliable results, I may perhaps be permitted to say that the explanation offered is apochryphal, but that the error indicated is about right. The radiator used at Allegheny was freely exposed, and the radiating surface of blackened copper suffered cooling by sudden gusts of wind, and also reflected some solar and sky radiation, because, although lampblack is a very poor reflector, the solar radiation so greatly exceeds that from a surface near the boiling-point of water that even this small reflection cannot be neglected. In my recent measures I have thoroughly protected the radiator from reflected sunlight and wind by a black cloth hood, and a suitable shelter.

With columns of moist air longer than 100 meters, the absorption increases very slowly. The case is similar to that of radiation of gases where, as I have shown in the work just cited, one meter of carbon dioxide is a maximum radiant layer. Lest anyone may suppose that this matter is doubtful, I will refer to a recent research on the "Nocturnal Variation of Temperature," by W. Schmidt, who gives in the *Sitzungsbericht* of the Wien Academy of Science for March 1909 a final value of the radiation coefficient of air, derived from meteorological observations, $10^4 \times \sigma = 1.84$, which may be compared with my value from laboratory experiments which gave 2.16.

On p. 172 of the *Annals*, the authors say: "It is highly probable that, considering the greater air-mass attending the oblique passage of many of the rays to space, nine-tenths of the radiation of the solid and liquid surface of the earth is absorbed by the water-vapor of the atmosphere even on clear days."

My value of the transmission of terrestrial radiation is four times as great as that of the *Annals*. The low angle of the rays makes little difference. An absorption which is already nearly complete for a zenithal ray, is not much increased in passing at a lower angle. Meanwhile, in spite of the absorption, there are regions of the spectrum between the bands which are passing through the air freely. This was first demonstrated beyond a doubt by my observation, in October 1884, at the Allegheny Observatory, of lunar radiation in the spectrum from a rock-salt prism between deviations

38° and 33°, corresponding to the radiation from bodies at terrestrial temperatures. This observation was afterward repeated many times by Mr. Langley and myself, always with the same result.

Further direct comparisons, not of an ideal "black body," as a passage on p. 174 of the *Annals* implies, but of the lunar surface and "some of the materials likely to be prevalent on the moon's surface," gave me for the effective midday temperature of the equatorial regions of the moon 454° Abs. C.,[1] which is 61° higher than the value which Abbot and Fowle assign by estimation, using their value of the solar constant. As I can see no reason for abandoning the result of my lunar measurement, I prefer to take it as a basis for an estimate of the solar constant, getting about 3 calories.

On p. 15 of the *Annals* the categorical statement is made that "we know that in general the lower layers of the air have smaller transmission coefficients than the upper ones, owing to the generally low level of the larger quantities of dust and humidity." The assertion, which is made without any reservation, and concerning solar radiation, is true only for the unsifted rays, and does not apply to the actual residual radiation which has already experienced its greatest absorption by aqueous vapor in the upper air. Layers of saturated or nearly saturated vapor have an additional and especially strong absorption. Such layers may be found at any altitude. The dust, it is true, is most prevalent in the lower layers of air, but its coefficient has not been considered separately in the *Annals*.

The coefficient of transmission of terrestrial radiation which I have given is supported by the data of meteorology. Percival Lowell, in his equation for cooling by nocturnal radiation,[2] establishes his thesis upon the fact that nocturnal cooling goes on two and one-half times as fast under a clear sky as below clouds. Taking 50 per cent for the transmission of telluric radiation by clear air, he obtains a "relative emissivity," $e=0.47$. Direct measurements having given me 40 per cent for the transmission by clear air, substitution in Lowell's equation with the following data:

[1] *Astrophysical Journal*, **8**, 284, 1898.

[2] "Temperature of Mars," *Proc. American Acad. of Arts and Sciences*, **42**, 661, March 1907.

cloud transmission $=2/5\times 0.40=0.16$, average for day sky $=0.28$, average day and clear night $=0.34$, average day and cloudy night $=0.22$, and using his data for absolute temperature and for cooling in centigrade degrees, gives

$$\frac{292-10}{292-4}=0.9196=\sqrt[4]{\frac{y(1-0.34e)}{y(1-0.22e)}},$$
$$e=0.585$$

This represents the radiated part of the loss, and if the true emissivity of the earth's surface is unity, 0.415 of the energy obtained from solar radiation is left to be dissipated by convection.

Lowell's figures are for temperate latitudes. In the very moist tropics, nocturnal cooling is only about half as great, while over the ocean the total diurnal change of temperature of the water is less than $\frac{1}{2}$° C. Eleven cloudy days in the tropics gave me for the nocturnal cooling an average of 2°.0, whereas sixteen clear days at the same station gave 5°.4 during the same interval. Mean temperature $=+30$° C. Assume transmission $=0.20$,

$$\frac{303-5.4}{303-2.0}=0.9887=\sqrt[4]{\frac{y(1-0.255e)}{y(1-0.165e)}},$$
$$e=0.185.$$

These examples show that whereas about 60 per cent of surface heat may be emitted as radiation from temperate regions, only one-third as much heat escapes in this way in the tropics. It is possible that the low value of 10 per cent, adopted in the *Annals* for the average transmission of telluric radiation, may apply to saturated air over the tropical oceans where the moisture is in an especially absorbent form; but this value cannot be admitted in the reduction of land observations in middle latitudes.

While the absorption of terrestrial radiation of great wave-length is overrated in the *Annals*, that of the solar radiation is undervalued. An allowance of about one-half of 1 per cent is made for all of the solar radiation of greater wave-length than 2.5 μ, "determined by consideration of the form of the energy spectrum of the 'black body' at 6000° absolute temperature" (*op. cit.*, p. 56). But this allowance for unobserved infra-red radiation is inadequate, and the mode of extrapolation is inadmissible, because, as I have

shown in my paper on "The Solar Constant," published by the United States Weather Bureau in 1901, the solar radiation is richer in long waves than the theory assumes, besides having suffered a large aqueous absorption which is relatively more effective here than for terrestrial radiation, since the solar curve is ascending rapidly throughout the region of great aqueous absorption where the terrestrial curve is descending to its limit.

The selective depletion of the shorter waves by the air may best be approached by considering the phenomena of sky light. The sky sends us one-fourth as much light as the sun, and for a middle altitude an equal amount is reflected by the air to space, or one-half of the sunlight received is dissipated in this way. This leaves a remnant of 0.67, since

$$0.67+(0.5\times 0.67)=1.005.$$

But the air-mass for altitude 45° is 1.42, which gives

$$(p)^{1.42}=0.67, \text{ and } p=0.75.$$

This coefficient of transmission corresponds to wave-length 0.57 μ in the lemon yellow; but the maximum luminosity for sky light is in the blue where the transmission-curve given in the *Annals* reads 0.67 instead of 0.75. Hence there must be further depletion of these rays in addition to the ordinary selective scattering which gives sky light.

The sources of sky light are complex. Though, in general, blue and violet are in excess in the light of the sky, there is a strong and wide absorption band in the violet whose limits I find from photometric measures are 0.405 μ and 0.460 μ, with the minimum at 0.433 μ. This band, which also appears in the spectrum of the direct rays, is attributed in the *Annals* to the abundance of solar lines near G. But though the wave-length of G is $\lambda=4308$ Å, and thus is not far from the minimum of the diffuse band, there is no extraordinary gathering of solar lines between the given limits sufficient to account for the band which represents an absorption of about 25 per cent. Consequently, the band is *not* of solar origin.

Now Edward L. Nichols has found (especially in high mountainous regions) that the sky spectrum has a variable *emission* band at wave-length 0.42 μ which, allowing for the rather vague

identification of the maximum point in his curves, may possibly have some connection with this band. My own observations, however, unlike those of Nichols, show the band as an *absorption* band in the sky spectrum. Further investigation will be needed to clear up these discrepancies, but the band is undoubtedly telluric and much more extensive than the gathering of solar lines near G. That a band or line should appear sometimes bright and at other times dark, that is, either as emission or absorption, is nothing new, although the controlling conditions, which are usually those of temperature, are not easily recognized if this band be such a case.

Nichols, it is true, found that the ratios of the spectral curves of sky luminosity to his "typical dawn curve" showed a *minimum* at 0.42 μ, but diminishing in strength as the sun rose higher, and succeeded by the aforesaid maximum in the middle of the day. My observation of the violet depression was made on the night sky illuminated by moonlight. This suggests that the "dawn curve" may be produced by selective scattering of shorter waves, in which violet predominates, and that the scattering is caused by a special material in the upper air which is in relatively strong illumination at the hour of dawn as compared with the deeper layers of the atmosphere. The latter furnish the absorption band, temporarily overpowered by the light from the upper layer, but reappearing soon after sunrise, when the upper light loses its early predominance; while later in the day the emission band again asserts itself in the sky spectrum at high altitudes through the continual formation of the special material in the upper air during the daylight hours.

The diversity of the phenomenon is further enhanced in the variability of the process of production of the special material which must be formed by ultra-violet sunlight, and must also be unstable, returning quickly to a normal and inactive condition, so that the active material becomes conspicuous only during that part of the day when the rate of its formation exceeds its decay. At low levels the emission band is liable to be neutralized in the sky spectrum by the absorption band present both by day and by night in the spectrum of the lower air.

Another observation by Nichols may possibly contain the clue to the substance which produces this band. He found that the emission band "appears regularly as the day advances in the mountainous regions of Switzerland during fine summer weather when there is a gradual formation of mist in the upper atmosphere. . . . . The maximum is commonly well established before noon and frequently persists until toward sunset."[1]

I suggest that the growth of this band is possibly connected with the formation of the complex water molecules, described in my *Atmospheric Radiation* (p. 100), to which Sutherland has given the general name of hydrols. Increasing ionization of the upper air by the sun's ultra-violet rays leads to the formation of nulcei of condensation, and this, combined with the midday upward movement of aqueous vapor at considerable altitudes, conduces to the formation of an exceptional amount of peculiar diffracting particles in the upper air during the middle of the day. The slopes of the logarithmic transmission-curves obtained by combining observations at high sun with others for a lower altitude of the sun have too low a slope, because the quality of the air has gradually changed in the middle of the day, and the transmission is overrated in a part of the spectrum where a small error makes a large difference in the resulting solar constant. The continual fluctuation of the solar radiant intensity through a wide range, shown by Crova's recording actinograph in the middle of the day with the clearest sky, is an evidence of the irregular distribution of an invisible obstructing substance which is most potent to deplete the solar rays when the sun is highest. Instead of rejecting low-sun measures, as has been done in the *Annals*, it might be better to reject those made near meridian altitude as being under suspicion to say the least.

If the wave-length assigned to the maximum of sky emission by Nichols could be shown to be correct, we might have further reasons for supposing that his band at 0.42 μ may be a hydrol band, because: (1) This wave-length agrees with that of a hypothetical band "No. 14" in extension of Kayser's diffuse series for aqueous vapor as shown in the series:

[1] Edward L. Nichols, "A Study of Overcast Skies," *Physical Review*, **28**, 126, 1909.

Band No. 6 (observed),	$1/\lambda = 1684$	
Band No. 9 (observed)	1833,	difference = 149
Band No. 11 (observed)	1978,	difference = 145
Band No. 12 (computed)	2118,	difference = 140
Band No. 13 (computed)	2253,	difference = 135
Band No. 14 (computed)	2381,	difference = 128

(2) The character of the band, which is diffuse and apparently irresolvable, agrees with that of the other members of the series. (3) The band seems to have some connection with the formation of mist, and, in general, depletion by selective scattering increases with the moisture in the air. Thus I have found from observations with the spectrobolometer that depletion of the shorter waves is greatest in summer. But this is equally indicated by the obvious fact that the red tints of sunset and sunrise are deepest in summer and especially in the tropics. Now the absorption lines of aqueous vapor are strongest at the *red* end of the spectrum. Their exceptional development should therefore increase the blue rays by comparison. On the contrary, it is the blue rays which are lacking, hence they have been depleted by something which increases with the abundance of water-vapor, but which is not water-vapor; and this substance, which appears to be most extensively formed out of aqueous vapor in summer, has a depleting action on the shorter waves which increases at a more rapid rate than the line absorption in the red. Thus the aqueous vapor does more than double duty, for it strikes out its own peculiar absorption bands in the red and infra-red, and also supplies a still more potent substance, depleting the rays of short wave-length.

Considering the nature of the depleting process, we find that the selective scattering of light by the atmosphere is a complex of diffraction and reflection from gaseous molecules, from condensation nuclei, and from dust. Incipient mist, apart from its own peculiar band absorption, increases the reflection of longer waves. The air of the tropical regions is especially loaded with material which produces scattering reflection, particularly that of the general, or nonselective sort. Ward says:[1] "The intensity of the light from tropical skies by day is trying, even almost unbearable." The

[1] *Climate Considered Especially in Relation to Man*, 88.

"pale whitish appearance" of the tropical skies is especially noted. The reflection from fine dust exceeds that from cloud. *Saturn's* rings are an example. They are composed entirely of discrete dust particles, and far outshine the clouded body of the planet (area for area). In consequence, the albedo of *Saturn* (0.78) is greater than that of the other major planets (0.65 to 0.75). The value which Abbot and Fowle give for the reflection of total radiation by the earth is 0.37. I have no hesitation in saying that this is too small. In my paper on "The Greenhouse Theory and Planetary Temperatures" in the *Philosophical Magazine* for September 1908 I have given 0.44 as a minimum value. The albedo, or light reflection, is larger. *Mars* has 0.27 and *Venus* 0.92. A mean of these, 0.60, is more likely than not to be exceeded for the earth, which resembles *Venus* more than it does *Mars*. A larger albedo means, of course, a smaller transmission of solar rays, and a larger value of the solar constant.

I wish now to pass to a meteorological argument which shows the necessity of a smaller coefficient of transmission for solar rays than Abbot and Fowle are willing to admit. If q is a quantity of heat, and dq the variation of this quantity on the unit of horizontal surface in the time dt, A being the solar constant, r the sun's distance from the earth, ζ the zenith distance of the sun, p the coefficient of atmospheric transmission, and ϵ the air-mass corresponding to a particular solar altitude, the general expression for the insolation is

$$dq = \frac{A}{r^2} p^{\epsilon} \cos \zeta \, dt .$$

Angot has summed this equation for all latitudes with various values of the average transmission, p_a, and Ward, in his work on climate just cited, gives curves of insolation for June 21, founded on these data, to which I have added a lower curve for $p_a = 0.25$ (Fig. 1). Now actually the distribution of temperature on the earth at the summer solstice agrees better with the lower curve, or even with one a little lower yet, as may be seen in the next figure (Fig. 2), where the curves should be congruous. The insolation curve is already a little too low on the left-hand side of the maximum, and yet it requires still further diminution on that side to

allow for the greater band absorption between latitudes 20° N. and 20° S. where the aqueous absorption reaches a maximum. I consider that the best value for the average transmission of solar rays is 0.18, as far as this can be inferred from the form of the curves. It must be remembered that this is an average for the entire sunlit hemisphere, because we have taken average tempera-

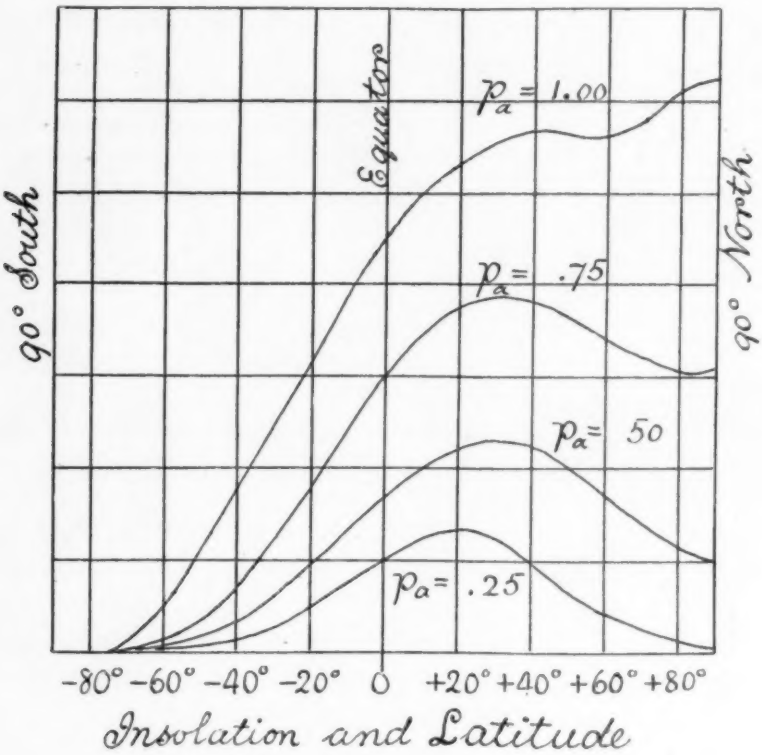


FIG. 1

tures. Such a transmission is entirely incompatible with a value of the solar constant as low as 2.1 calories. It requires a value greater than 3 calories, but perhaps not much greater.

Let us suppose, for illustration, that A, the solar constant, is 3 cal. per sq. cm per min., p, the coefficient of transmission obtained by considering the depletion by selective scattering alone, is a

factor = 0.6, and B, the lost radiation, is 0.6 cal., equivalent to a coefficient of absorption of 0.2. Then

$$Ap - B = (0.6 \times 3) - 0.6 = 1.2 = R,$$[1]

and

$$(R+B) \div p = A.$$

Now suppose that A is unknown, and that there are many fine absorption lines which escape measurement by the spectrobolometer,

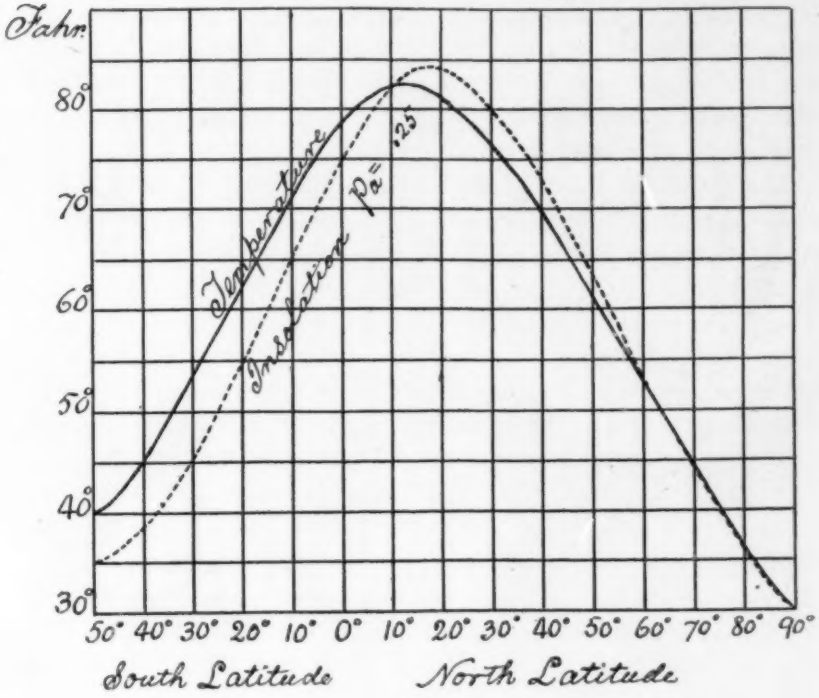


FIG. 2

or that there are dust particles so large and opaque that they absorb a part of the radiation, so that B is estimated to be 0.2 cal., or $6\frac{2}{3}$ per cent of the original radiation, instead of 0.6 cal., or 20 per cent. Then

$$\frac{R+B}{p} = \frac{1.2+0.2}{0.6} = 2.33 \text{ cal.}$$

[1] The bands do not much affect the region of the spectrum in which selective scattering occurs. Hence, although the processes are simultaneous, they are best treated as if they were separate in such an approximate computation as this.

If p is also overestimated and presumed to be 70 per cent, instead of 60 per cent,

$$\frac{R+B}{p}=\frac{1.2+0.2}{0.7}=2.00 \text{ cal.}$$

It seems to me that this is exactly what has been done in these deductions of a solar constant by inadequate methods. The depletion of the incoming radiation has been underestimated. Too low an effective temperature has been assigned to the earth's radiating layer in consequence, and it has been necessary to assign too small a value to the atmospheric transmission of terrestrial radiation in order to reconcile data which are incompatible.

Too little account is taken in the *Annals* of a significant feature of the diurnal variation, namely, the apparent increase of transmission for large air-masses with a low sun. The frequent departure of the logarithmic curves of atmospheric transmission from straight lines is dismissed as being without special significance. The air layers are presumed to differ from each other in transparency, but only by a gradual progression. But the actual atmosphere contains ingredients which are sharply confined to definite layers with distinct limits. Of two stations, one at a low level and the other near the upper limit of the dust-bearing air, the more elevated experiences but a small fraction of the depletion of radiation by the dust which is suffered by the lower station, even though the air-masses may not differ much. The dust factor must be treated separately from the others. It has usually been completely ignored.

The flat-topped diurnal actinometer-curves, recorded by Crova's registering actinometer, are a record of the disappearance of a large amount of radiation in the middle of the day. The aqueous origin (whether direct or indirect) of this feature is shown by its partial disappearance in cold dry weather. There is a lack of symmetry in the curves obtained on mountains, the morning values of the insolation being higher than those obtained in the afternoon with the same altitude of the sun. This also is to be connected with the increase of the tension of aqueous vapor in the upper air in the afternoon. These details show that in every way the absorption of the incoming solar rays has been underrated.

It has been repeatedly asserted that the temperature of *Mars* must be far below the freezing-point, and this in spite of the obvious melting of polar snows, and the determination of the amount of water-vapor in the Martian atmosphere, which gives a dew-point not far from freezing temperature,[1] and of course a dry-bulb temperature considerably above the freezing-point. These facts point to a solar constant of 3 calories or more.

Even our earth, according to Abbot and Fowle, has an effective temperature of only $-10°$ C., and hence must be assumed to radiate from an upper layer of frosty air, since the surface is so much warmer. I present a revision of this estimate, continuing my illustration of a hypothetical constant of 3 calories. After subtracting the lost radiation, 0.6 cal., I take the mean radiation received by a half-clouded earth = 2.4 cal. per sq. cm per min. before application of the transmission coefficient. The average transmission of the sifted remnant corresponds very nearly to that for a solar altitude of 27°, for which, assuming $p = 0.56$,

$$p^s = (0.56)^{2.2} = 0.279.$$

Radiation directly received from the sun,

$$R = 2.4 \times 0.279 = 0.67 \text{ cal.}$$

to which 10 per cent is to be added for sky reflection, making 0.74 cal. In emission this energy is distributed over a doubled area, becoming

$$\text{Emission} = 1/2 \times 0.74 = 0.37 \text{ cal./sq. cm min.}$$

In considering the radiant temperature we shall do well enough to confine the computation to what goes on with a clear sky. The mean temperature of the earth is about 288° Abs., and its emissivity is not far from unity. Unobstructed radiation to space should give by the fourth-power law

$$0.523 \text{ cal./sq. cm min.}$$

Only 0.4 of this radiation can penetrate through the clear air, as I have shown (*ante*, p. 375). This gives for radiation directly escaping:

$$0.4 \times 0.523 = 0.209 \text{ cal./sq. cm min.}$$

[1] *Lowell Observatory Bulletin*, No. 36, 1, 211.

The surface receives 0.370 cal., and the difference

$$0.370-0.209=0.161 \text{ cal./sq. cm min.}$$

is transferred to the air and emitted at a temperature of 215° Abs. $=-58°$ C. But this is almost precisely the temperature of the upper static layer which is truly the radiant layer of the atmosphere, since convection has ceased, and the heat received can be dissipated only as radiation.

The abandonment of low-sun observations because low-lying mists render such measures uncertain, and the selection of midday observations on the plea that the sky is then clearer, is at first sight plausible; but an examination of the diurnal curves of insolation obtained by the actinograph shows that the midday observations are the most irregular ones. If any observations are to be rejected, it should be these, subject as they are to continual variations and to the suspicion of a large constant error which is reduced to a practical certainty by the arguments which have been presented as well as by others which are to follow. The midday observations need not be absolutely thrown away; but we can recognize that, taken by themselves, they are misleading, that they require large corrections, and that the constant fluctuations of solar radiation in the middle of the day with the clearest sky, point to the presence of invisible "clouds" of absorbent vapor, which are more dangerous than low-lying mist at the time of low-sun observations. The mist, if it exists at all, is visible; but the diurnal variation in the transmissive quality of the upper air has to be inferred from the observations themselves. The mists of the morning may compensate for the midday diminution of transparency of the upper air, and comparisons of actinometer readings taken at morning and at noon may sometimes yield transmissions near the truth. Apparent transmissions founded on afternoon series of actinometer readings are usually too large, owing to the change of atmospheric quality. Such measures should be tested by carefully timed observations at the lowest observable altitude of the sun, which will at least show whether the measurements are capable of reduction. They usually demonstrate that the absorbent quality of the medium has changed.

If spectrum measures are used, it is found that the blue and violet are subject to extraordinary variations which are not yet entirely understood. The yellow and red are thickly strewn with atmospheric lines. In the intermediate green, however, there is a region sufficiently free from these disturbances to yield an approximate transmission coefficient if widely different air-masses are compared.

Observations covering a limited range of air-mass may be in apparent agreement. The real test is to see whether the agreement continues when the range is extended. The almost total absence of low-altitude records in the *Annals* is to be regretted; and still more unfortunate is the dependence which the authors have placed on summer observations almost to the exclusion of winter work, although the absence of moisture is of the greatest importance in securing a reliable measurement of the solar constant.

Langley's *bon mot:* "That is the *best* value of the solar constant which is nearest to three calories," may be more than a bit of humor; but, on the whole, the indications point to a value greater than three. A value of the solar constant not less than three, nor more than four, calories, accords with the data of meteorology. It also satisfies the demands of the best measures of nocturnal radiation and of the transmission of telluric radiation, agrees well with the temperature of the sun inferred from Wien's formula with Paschen's constant, as modified by Lummer and Pringsheim, and from the actual temperature at the focus of a large mirror, requires no modifications of the methods of evaluating the transmission of direct solar rays which are not abundantly demonstrable as requisite, and in every way harmonizes a great body of facts which would otherwise be found to be in conflict.

WESTWOOD, MASS.
July 1911

THE PRODUCTION OF LIGHT BY CATHODE RAYS

By GORDON S. FULCHER

In a previous paper[1] it was shown that light is emitted as a result of the collision of canal rays with gas molecules and that its intensity is proportional to the mean energy transmitted times the number of collisions. An analysis of the Doppler effect seemed to indicate that the neutral rays are far less effective than the charged rays in producing this luminescence. Since then an attempt has been made to separate the charged from the uncharged rays by means of a magnetic field and to examine the light-producing effect of each; but, in agreement with recent results obtained by Wien and Koenigsberger, this was unsuccessful. Ionization and neutralization produced by the collisions with gas molecules and corpuscles, introduce charged and uncharged rays into the deflected and undeflected beams, so that, with the apparatus used, a very strong magnetic field reduced the number of charged rays in the undeflected beam by only about 10 per cent.

The phenomena occurring in the region traversed by canal rays are fewer than in the main discharge, but nevertheless are quite numerous. The beam itself consists of both atoms and molecules; some are positively charged, some are neutral, and some negatively charged. Ionization, neutralization, dissociation, and association are continually taking place; secondary cathode rays are being shot out of the ionized atoms producing more ionization in their turn; and in connection with each one of these transformations light may be emitted. As the attempt to separate neutral and charged rays had failed, I turned my attention to the secondary cathode rays, since these are identical with cathode rays of the same velocity and can be most easily separated from other kinds of rays.

Only two papers dealing definitely with the subject have been found. Wüllner,[2] in a brief note, says that he carefully separated

[1] *Astrophysical Journal*, **33**, 28, 1911.

[2] *Physikalische Zeitschrift*, **1**, 132, 1899.

cathode from canal rays, though he gives no further description of his apparatus, and reports the following results: In hydrogen, the cathode rays produce the compound line spectrum, wrongly termed the "band spectrum." In oxygen, they produce the cathode bands at $\lambda\lambda$ 5980, 5600, 5260 and the triplet 5330, but no spark lines. In air, the results were inconclusive. Lewis[1] concluded from a comparison of the spectra of the light from various parts of the discharge in air that the cathode rays cause the emission of the negative bands of nitrogen. He also investigated the luminescence produced by cathode rays in certain metallic vapors.[2] Because of the importance of the matter, an attempt has been made to verify these conclusions. Some crude preliminary results obtained with the apparatus described below may be of interest.

APPARATUS

The anode *D* (Fig. 1) was made of soft iron 1 mm thick in the central portion, and through it a hole (*H*) 0.020 cm in diameter was drilled. The cathode (*C*) was arranged so that its distance from the anode could be varied by turning the key *S* which fitted into a conical ground greased joint. The space behind the anode was inclosed by a brass cylinder *E*, except for the opening *W* which allowed light from the beam emerging from *H* to reach the single-prism spectrograph. The fine beam itself was adjusted so as to be at the focus of the collimating lens *L*, and thus took the place of the slit. It was found—what is doubtless well known—that when the prism was turned from the position of minimum deviation so as to decrease the angle of incidence, the width of the image of a wide slit increased more rapidly than the dispersion; but that when the prism was turned so as to increase the angle of incidence, the width of the image decreased more rapidly than the dispersion, so that lines could be separated in this position which would overlap with the prism placed for minimum deviation. By turning a large 45° prism, having a length of 10 cm at the base of its faces, so as to increase the angle of incidence as much as possible, the width of the image was reduced to but half that of the slit, while the dispersion

[1] *Astrophysical Journal*, **17**, 258, 1903.

[2] *Ibid.*, **16**, 31, 1902.

was not much less than with the prism placed for minimum deviation.

A soft iron pole-piece *P* was fastened inside *E* and a weak electromagnet *M* placed just outside the tube. The discharge chamber (*I*) and observation chamber (*II*) could each or both be connected to a reservoir of gas through a fine capillary tube and also to a Gaede pump. Because of the smallness of the hole connecting the two chambers, the pressure in either chamber could be kept several times greater than that in the other by connecting it to a capillary and the other to the pump; or the pressure could be maintained the same in both. Usually a continuous current was secured by the arrangement described in the previous paper (*op. cit.*)—a condenser kept charged by an induction coil and allowed to discharge continuously through a high resistance in series with the discharge tube.

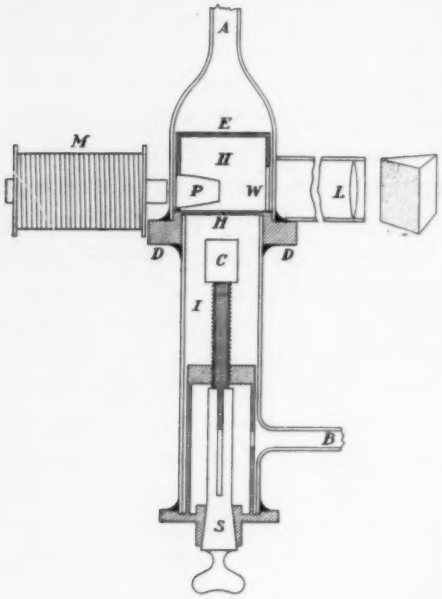
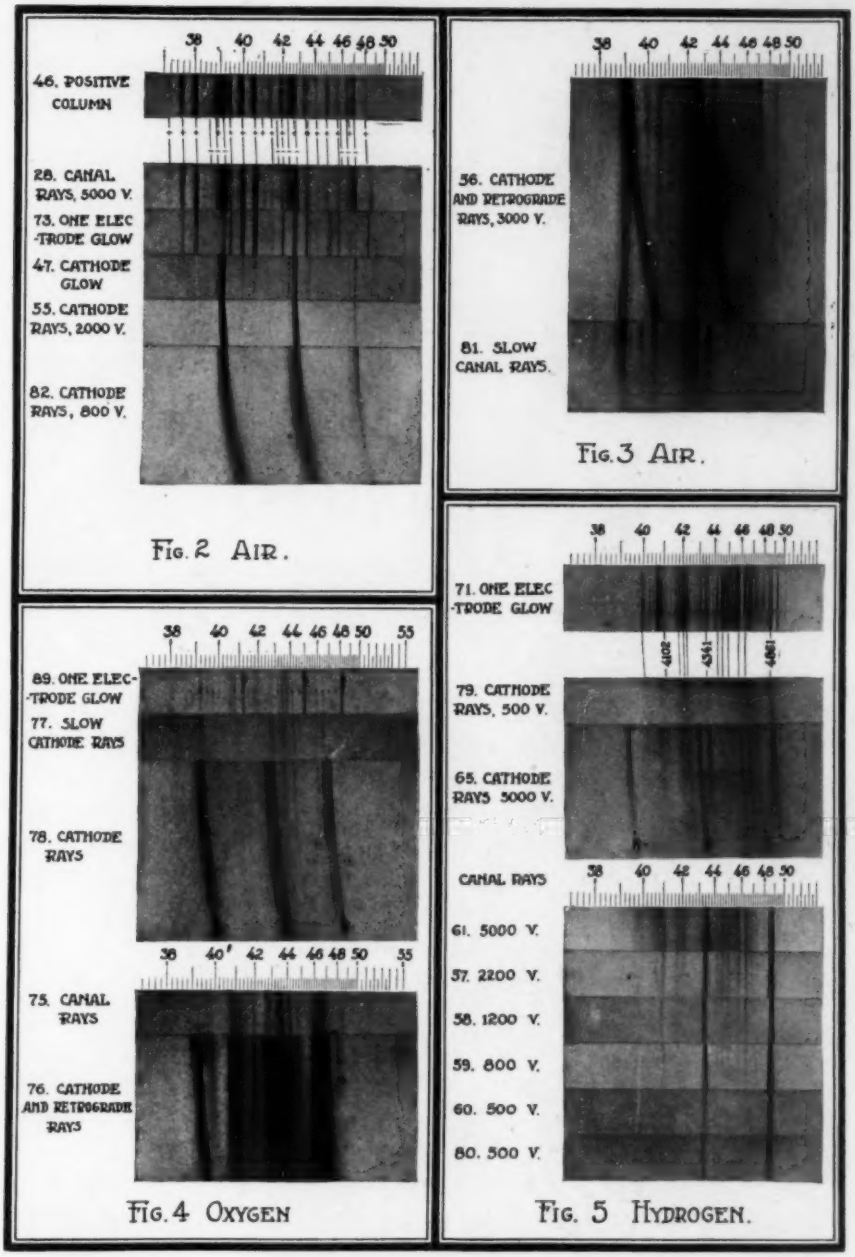


FIG. 1

RESULTS

Air.—Fig. 2 (Plate XVII) shows reproductions, enlarged 5 times, of a number of spectrograms of the nitrogen bands, made under various circumstances. Nos. 55, 56, 81, and 82 were taken with the apparatus described above. The spectrum of the deflected beam is seen to consist solely of the negative bands, in agreement with Lewis' conclusion. No. 82 is badly overexposed (12 hours), yet shows only a trace of the strongest positive bands. Compare also Nos. 65, 76, and 78. In the case of 56, the arrangement was such that retrograde canal rays as well as cathode rays passed through

PLATE XVII



THE PRODUCTION OF LIGHT BY CATHODE RAYS

the hole in the anode. The former produced the undeflected lines, the latter the deflected lines. The line marked with two crosses is a scratch on the negative. The difference between the spectra due to the canal and cathode rays is thus strikingly shown.

It was thought that the spectrum due to very slow canal rays might also be simpler. However, very slow canal rays are quickly dispersed, have a short range, and also produce very little light. No. 81 was taken with an ordinary discontinuous discharge, 1000 volts, and shows both positive and negative bands. I hope to try much slower rays with a better apparatus.

It was noticed that when the vacuum was such that the discharge passed with difficulty between the two electrodes, a luminescence appeared in the tubes leading from the discharge tube to the gas reservoir and pump. The spectrum was photographed and showed the positive bands much more strongly developed than the negative, No. 73. When one electrode of a Geissler tube was connected to the high-potential electrode of a tube through which a high-potential discharge was passing, a glow appeared in the Geissler tube whose spectrum was identical with that of the luminescence in the connecting tubes. This will be referred to as the one-electrode glow discharge.

Hydrogen.—In hydrogen, the cathode rays produce both the compound line and the main series spectrum, Nos. 79, 65; but the latter is relatively much less intense than in the light due to canal rays, and less intense in the spectrum produced by slow cathode rays, No. 79, than in that caused by the faster rays, No. 65. In the spectrum of the one-electrode glow discharge, No. 71, the lines of the main series are no stronger than neighboring strong lines of the compound spectrum.

From the series of spectrograms of light produced by canal rays, Nos. 57 to 61, it is clear that the compound spectrum is relatively less intense for the slower rays. The intensity of the light in the case of the slowest rays (500 volts) was so small that both plates 60 and 80 are underexposed in spite of long exposures (20 hours), so that they do not prove the entire absence of the compound spectrum under those circumstances; but it seems probable that in the case of sufficiently slow canal rays the main series line-spectrum of hydrogen is the only one excited.

Oxygen.—All the spectrograms obtained show the elementary or spark spectrum lines of oxygen, even that produced by slow cathode rays. The main spark lines are marked by dots on No. 89. The crosses indicate *CO* bands. Also the main series lines λλ 3947, 4368 seem present on all the spectrograms. The first subordinate series line λ 4773 appears on Nos. 75 and 89, the line λ 4968 on the other three, Nos. 76, 77, 78. The second subordinate series lines λλ 4802, 5020, 5435 are faintly visible on No. 76. The cathode bands λλ 5260 and 5580 are present in Nos. 76 and 78; they show no undeflected streak in No. 76. All the lines appearing in No. 76 seem to have a deflected branch, certainly the spark lines, and probably also the series lines. Properly to study the behavior of the various spectra of oxygen, an apparatus sensitive to λ 5600 and having greater dispersion is necessary. The spectrograms Nos. 76, 77, 78 represent exposures of about 20 hours each.

Summary.—The results described above show that the spectrum of the light produced by cathode rays (1) in nitrogen consists solely of the negative bands; (2) in hydrogen consists chiefly of the compound spectrum together with the main series lines, relatively weak; (3) in oxygen consists of the negative bands together with the spark lines and series of triplets. They agree in general with the results of Wüllner and Lewis summarized above.

Also it seems probable that slow canal rays in hydrogen produce only the main series lines.

Discussion.—In this preliminary note, I shall not attempt to explain the numerous observations regarding the variations in the relative intensity of the different spectra of the above gases under varying circumstances which have been reported, but shall merely indicate how this line of approach promises to afford definite information as to which atomic system emits each spectrum. Consider the case of cathode rays passing through a diatomic gas. They will collide with the molecules and may conceivably ionize them either (1) by shooting out an electron from one of the atoms of the molecule without causing dissociation or (2) by hitting between the two atoms so as to render the molecule unstable, causing dissociation as well as ionization. If we add neutralization, recombination of the ionized molecules, and negative ionization by the attachment of a stray corpuscle to a molecule, we have probably listed all the pos-

PLATE XVIII

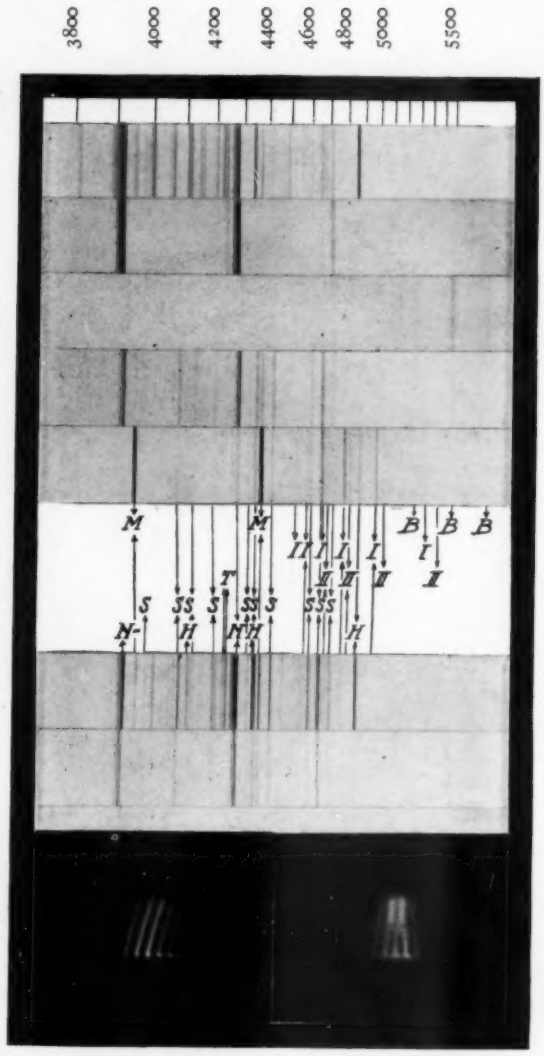


FIG. 7

136. Air-Canal Rays

117. Air-Cathode Rays (3000 v.)
λλ 3914, 4278, 4709

120. *O*-Cathode Rays (3000 v.)
Trichromatic Plate

121. *O*-Cathode Rays (3000 v.)
Seed 27 Plate

133. *O*-Diffuse Canal Rays (1500 v.)

B, Negative Bands: λλ 5260, 5600, 5860
M, Main Series: λλ 3947, 4368
I, 1st Sub. Series
II, 2d Sub. Series
S, Spark Lines *T*, Triplet, λλ 4223, 4233
N, Nitrogen *H*, Hydrogen

131. *O*-Canal Rays (5000 v.)

121. *O*-Cathode Rays

FIG. 8

(*a*) Cathode Rays (*b*) Diffuse Canal Rays

sible transformations taking place in the space traversed by the cathode rays. Because of the relatively long mean free path of molecules at that pressure, the last three transformations will not be confined to the path of the rays, but will be distributed throughout a considerable space around it; whereas the direct ionization by the cathode rays must take place largely in the path of the beam (since the scattered rays will be slower, less concentrated, and less effective). Hence it is reasonable to assign to the two ionizing transformations most of the light photographed with the above apparatus. In the case of very slow cathode rays it seems probable that ionization without dissociation alone may be possible or may at least largely predominate. If, then, the light produced by very slow cathode rays consists solely of a band spectrum, that spectrum is probably emitted by the positively charged molecule of the gas in question.

In the case of nitrogen, slow canal rays produce only a band spectrum; that is, no dissociation is produced. The difference between the band spectrum produced by canal and cathode rays may be due to the difference in the method of ionization because of the difference in size and in the electric field around the two kinds of rays. Hence the vibrations aroused, though not unrelated, are different.

September 4, 1911

AN ADDITIONAL EXPERIMENT

Since the above was written, a *new apparatus* has been made which gives more light, permitting the use of a focusing lens and a slit. The cathode is the same as before, but the anode is changed as shown in Fig. 6. There are several parallel slits cut in it, 1 mm wide and 1.5 mm apart. A comb containing a similar set of slits is placed 5 mm back of the anode so as to intercept any rays which pass through the anode in a direction normal to its front surface. Two magnetic pole-pieces, 3 mm apart, are so placed that when the field is adjusted to a suitable strength, cathode rays passing through the slits in the anode are deflected so as to pass through those in the comb, while the less deflectable atomic rays are still intercepted.

A photograph of the deflected bundles of cathode rays, viewed edge-on, is shown in Fig. 8 (*a*), Plate XVIII. The apparatus is

not as successful as had been hoped in eliminating atomic rays from the chamber back of the anode for a wide range of cathode-ray velocities. It was found that when the terminals are reversed so that the anode becomes the cathode and canal rays pass through the slits, the rays are not entirely intercepted in the case of low velocities; or rather, diffuse secondary canal rays, created by collision with gas molecules, produce luminous undeflectable bundles,

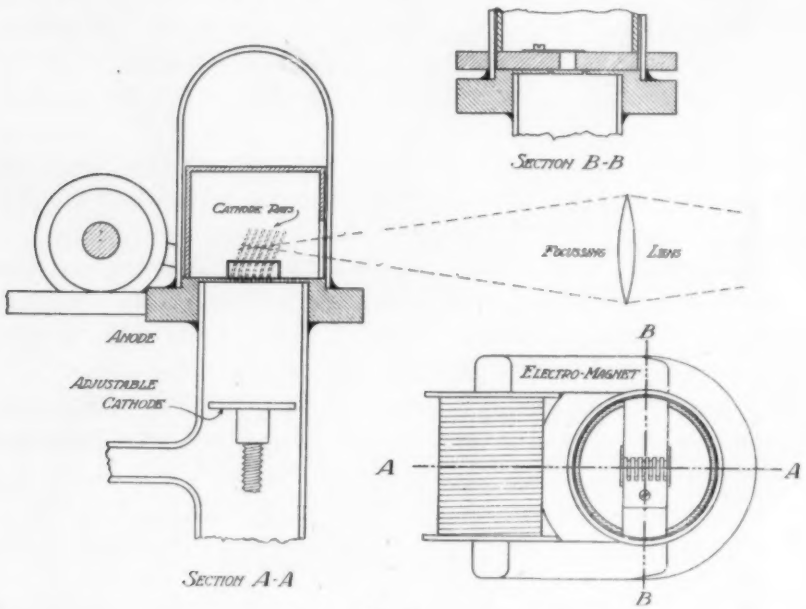


FIG. 6

which are shown photographed in Fig. 8 (*b*). However, with a cathode-fall of 2000 volts or more, quite pure bundles of cathode rays can be obtained, as is shown by the fact that, when the magnetic field is zero, the chamber back of the anode is practically dark. Yet the range of velocities which can be tested with this apparatus is unfortunately very limited. This disadvantage is partially offset by the opportunity, unexpectedly afforded, to obtain the spectrum of the light produced by the slow diffuse canal rays, which turns out to be of some interest. But with this apparatus, it must be kept in mind that we analyze the light from the region traversed by the rays

and not merely the light due to the primary action of the rays as with the previous apparatus.

The *results* (Plate XVIII) confirm and supplement those reported above.

Air.—Cathode rays cause the emission of the negative nitrogen bands, $\lambda\lambda$ 3914, 4278, 4709. No. 117 shows merely a trace of the nitrogen positive bands and no oxygen lines. An exposure twelve times as long (100 hours) would be necessary to bring out the oxygen cathode-ray spectrum as shown in No. 121, which required an exposure of 24 hours. *Diffuse canal rays* produce (primarily and secondarily) the same spectrum as slow canal rays—positive and negative bands both strong.

Hydrogen.—Cathode rays produce both the compound line and the series spectrum, but the latter is relatively very much weaker than in the light produced by canal rays. *Diffuse canal rays*, like slow canal rays, cause the emission of both spectra (directly and indirectly), but the series spectrum is much the stronger. As Stark and Hermann[1] have found, the spectrum of the light from the canal ray region shows the series spectrum relatively more intense the greater the velocity of the rays. This seems a direct contradiction to the statement made above (p. 391) that slow canal rays produce only the series spectrum in hydrogen; but when we consider that we analyze in one case light from the whole region, including that due to secondary ionization, and in the other case only that due to the primary action of the canal rays in the path of the beam, the contradiction is seen to be easily explicable and very significant. The primary luminescence depends directly on the energy of the rays, while the secondary luminescence, due to secondary cathode rays, is, within certain limits, independent of the velocity of the rays. Thus the greater the energy of the rays, the more the primary will predominate over the secondary luminescence. When, however, the beam itself is used as a line source of light so that the secondary luminescence scattered throughout the surrounding region is relatively ineffective, the spectrum of the light due to the primary action of the rays is obtained. It seems probable, therefore, that the series line spectrum of hydrogen is emitted as a result of the collision of canal rays with molecules;

[1] *Physikalische Zeitschrift*, **7**, 95, 1905.

the compound line spectrum as a result of the ionization of hydrogen molecules by cathode rays.

Oxygen.—The spectrum due to *cathode rays* consists chiefly of the negative bands, $\lambda\lambda$ 526, 560, 586, but spark lines and series triplets are also present, though faint. No. 120 was made on a Seed No. 27 extra-rapid plate, 24 hours' exposure; in the case of No. 121, a Cramer Trichromatic plate was used, 30 hours' exposure. In the spectrum due to *diffuse canal rays* (No. 123), the main series lines, $\lambda\lambda$ 4368 and 3947 (see Runge and Paschen, *Wied. Annalen*, **61**, 641, 1897), are remarkably strong; the two subordinate series are well developed, while the spark lines are relatively weak. With fast *canal rays* (No. 131), the spark lines predominate, while slower canal rays give rise to spectra intermediate between No. 131 and No. 123. The variation in the relative intensity of the various groups of lines when produced in various ways, is shown roughly in the following table. The intensities of the various lines on the plates were estimated, and the average intensity was computed for the spark lines (S), $\lambda\lambda$ 3974, 4072, 4186, 4318, 4348, 4415, 4591, 4642; for the first subordinate series triplets (I), $\lambda\lambda$ 4773, 4968; for the second subordinate series triplets (II), $\lambda\lambda$ 4673, 4802. Then the scales were adjusted so that the average for the spark lines came out the same for all. No trace of the line λ 3955 nor of the series of doublets was found (cf. Runge and Paschen, *op. cit.*).

Oxygen	Diffuse Canal Rays (1500 v.)	Canal Rays (2000 v.)	Canal Rays (5000 v.)	Cathode Rays (3000 v.)
Negative Bands (B) (No. 121)	—	—	—	20
Spark Lines (S)	4	4	4	4
λ 4368	40	12	4	4
λ 3947	25	8	2.5	1
First Sub. (I)	6	3	0.5	0.4
Second Sub. (II)	2.5	1.3	0.3	0.2

As a result of the discussion above on p. 393, it was concluded that when cathode rays produce a band spectrum it may reasonably be assigned to the positively ionized molecule of the gas in question. The negative bands of nitrogen and the negative bands of oxygen, then, are probably emitted by the respective molecules as a result of ionization by cathode rays.

UNIVERSITY OF WISCONSIN
November 12, 1911

EFFECTS OF VARIATIONS OF VAPOR-DENSITY ON THE CALCIUM LINES H, K, AND *g* (λ 4227)

By OLIVER J. LEE

The element calcium has from the beginning played an important part in astrophysics. Two of its principal spectrum lines, H, λ 3968, and K, λ 3934, have been chiefly used by solar physicists in the study of the sun with the spectroheliograph. The importance of these lines is being further enhanced by their growing significance in stellar spectroscopy. There arises the question of the interpretation of strong, sharp lines of calcium in a stellar spectrum when accompanied by broad lines of hydrogen, helium, and other elements. Radial velocity observations of stars of the spectroscopic binary type have in various cases shown an apparently anomalous behavior for these lines.

Reference to Kayser's *Handbuch der Spectroscopie*, V, 235, shows that no less than 130 articles have been published which are concerned wholly or in part with the spectroscopic treatment of calcium.

The well-known experiment of Sir William and Lady Huggins on vapor-density effects in the spectrum of calcium, as well as the recent researches of King, St. John, and others, have suggested the desirability of trying to isolate one of the variables in the production of the calcium spectrum and to govern its variation, while keeping the others approximately constant. The present paper gives the results of an attempt to get quantitative data in the case of varying vapor-density, and limits itself to determining the relative vapor-densities at which self-reversal occurs in the lines H, K, and *g*.

Methods used to determine vapor-density constants for elements that volatilize at low temperatures, such as sodium and sulphur, are obviously unsuitable for calcium. Owing to the high temperature required, about 2500° C., it was out of the question to confine any given amount of vapor and still be able to get its light. Instead,

the rate of volatilization of the element under standard conditions was assumed to be constant and was made the basis of the experiment. Thus it was assumed that if twice as great a surface area of calcium was exposed in one case as in a second case, the density of the vapor after a given time of heating would be twice as great in the former instance as in the latter. The conditions for each observation therefore became:

1) Constant temperature sufficiently high to volatilize calcium freely;

2) Volatilizing areas of calcium which should be proportional to given predetermined ratios;

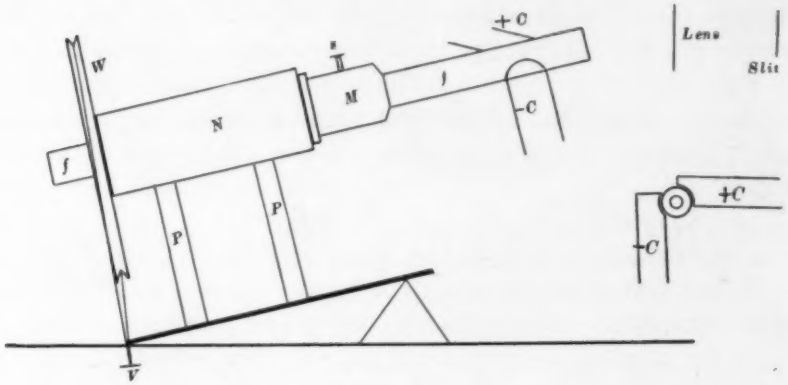
3) Exposure-times short enough to eliminate as much as possible inevitable changes of conditions.

APPARATUS

The apparatus employed consists of a rotating carbon heating tube in which calcium is vaporized and a concave grating spectrograph.

The heater is a carbon rod, *ff*, 1 cm in diameter, which has a hole in one end 5 mm in width and 15 mm deep. The carbon is mounted in the hollow spindle *M*, to which it is fastened by means of the set-screw *s*. At the other end of the spindle is fastened a light grooved wheel *W*, 8 inches in diameter, which is connected to the small pulley of a motor, *V*, by means of a stout cord. Spindle and wheel are mounted in the brass bearing *N*, and this is rigidly bolted to the cast-iron base of the arc apparatus by means of the steel rods *PP*. The carbon-tube was made to rotate smoothly about 100 times a minute between the poles of an arc, each a carbon rod 13 mm in diameter. This arrangement practically gave the heating effect of two arcs, the rotating heater being a negative pole on the upper side and a positive pole on the lower. A stationary heater similarly mounted was first tried, but in that case the negative pole, *C*, burned its way through the thin wall of the tube so quickly that no reliance could be placed upon the temperature within. With the rotating heater this wear took place so slowly that the temperature could be kept comparatively constant for the 2 to 4 minutes during which each was in use.

The current used throughout was 20 amperes at 40 volts. This produced a temperature of about 2500° C. After finding a radiation pyrometer impracticable for such a small heated surface, and an electric thermo-couple inadequate, the heat within the tube being actually greater than was expected, the temperature given above was determined indirectly. Several substances whose melting points are established, such as salt, 800° C., iron, 1503°, platinum, 1753°, and iridium, 2350° ±, were introduced into fresh heating tubes and the current was slowly increased until they melted. The currents were plotted against melting temperatures and the required temperature was read from the resultant curve.



By means of a 3-inch lens the light from the center of the tube was projected upon the slit of the spectrograph, the dispersion piece of which is a Michelson concave grating of 2 meters radius. The ruled surface is 50 mm×82 mm and has 590 lines to the mm. In practice, with a fairly wide slit, two good lines 0.3 Å units apart were well resolved. The plate was held in the camera, curved in an arc of 1 m radius, and 7 exposures could be made on each plate without opening the camera.

METHOD OF OBSERVING

Two distinct methods have been used in this experiment to govern the relative amounts of calcium introduced into the various tubes.

1. A supersaturated solution of calcium chloride was made by taking equal parts by weight of distilled water and the salt. Call this standard solution A. By means of glass droppers, solutions of calcium strength A, A/2, A/4, A/8 . . . . A/128 were introduced into tubes of the adopted standard size, 5×15 mm. Series of such tubes were filled and left in an upright position to evaporate part of the water and to let the salt crystallize on the walls and bottom of the carbon bore. When the tubes were dry, they were placed in the rotating spindle and slowly heated until the water of crystallization was given up without explosive scattering of the material. From 3 to 7 exposures of 30 seconds each were made with every tube so that its photographic record might be complete. The normal plate first shows the calcium lines weakly, then as the heater gets its proper temperature and vaporization of the calcium is rapid the lines increase in strength to a maximum for that tube.

It was found that this method gave consistent results for the lower densities, the ratio of which could be determined with considerable accuracy. For the greater densities required to reverse H and K, however, the large amount of material in the tube made the vaporization more or less uncertain and the resulting reversals were in consequence erratic. The second method was found to give more satisfactory results for these higher densities.

2. Metallic calcium was sawed into very thin rectangular pieces $\frac{1}{2}$ mm thick and 3 to 4 mm wide. These were clipped across into slivers fairly uniform in size. The slivers, weighing on an average 2 milligrams each, were weighed out into charges of from 5 to 100 milligrams. After tipping the apparatus as shown in the diagram the tube with its charge was fastened in the spindle and slowly heated while revolving. As the chamber walls became heated the metallic particles would spread themselves uniformly over them and adhere while vaporizing. Multiple exposures were made as in the first case. With neither method was a tube used for more than one charge. About 800 photographs were made with 225 tubes.

The following tables give typical data for reversals with change of vapor-density. The first gives results obtained by method (1) and covers the low order of vapor-density required to reverse *g*.

Calcium Charge	Number of Exposures	Results
A/4.....	20	*g* reversed on all, K on 2, H on 1
A/8.....	13	*g* reversed on 10
A/16.....	14	*g* reversed on 8
A/32.....	20	One doubtful reversal of *g*

The following table gives results obtained by method (2) covering the range for H and K reversals:

Calcium Charge	Number of Exposures	Times Reversed		
		g	K	H
0.100 gram.....	33	33	27	16
0.050 gram.....	33	33	22	10
0.025 gram.....	31	28	7	0
0.010 gram.....	32	22	1	0
0.005 gram.....	29	9	0	0

Comparing the reversal times with calcium charges A/16, 0.010 gram and 0.005 gram, it is seen that an A/16 charge of calcium chloride corresponds in this experiment to one of about 0.0085 gram of metallic calcium.

CALCIUM IMPURITIES IN CARBONS AND IN AIR

An attempt was made to purify the carbons used for heating tubes by repeatedly raising them to a temperature of several hundred degrees in a strong atmosphere of chlorine and then boiling them in distilled water. The desired effect was not obtained, for there seemed to be no diminution in the amount of calcium present. The reason for this failure is explained by an experiment by W. N. Hartley, reported to the Royal Society, May 11, 1911, in which he shows that there are copper and calcium impurities in the air in sufficient quantities to give the characteristic spectrum lines of these elements. As all of my exposures were made with the arc in air, even as radical a treatment of the tubes as that described would of course not eliminate the calcium lines. The order of magnitude of the impurities in carbons and air was found by noting at what strength of calcium solution the lines should begin to show

an increase over their intensity when the carbon only was used. No increase was visible until a charge of A/100 was reached. At A/50 the lines were estimated to be twice as strong as those derived from the carbon alone in air. From this it was concluded that the calcium impurity in the carbon heaters and the air together did not exceed 2 per cent of an A charge, and that its effect on the ratio A to A/8, for example, was negligible.

APPEARANCE OF THE LINES

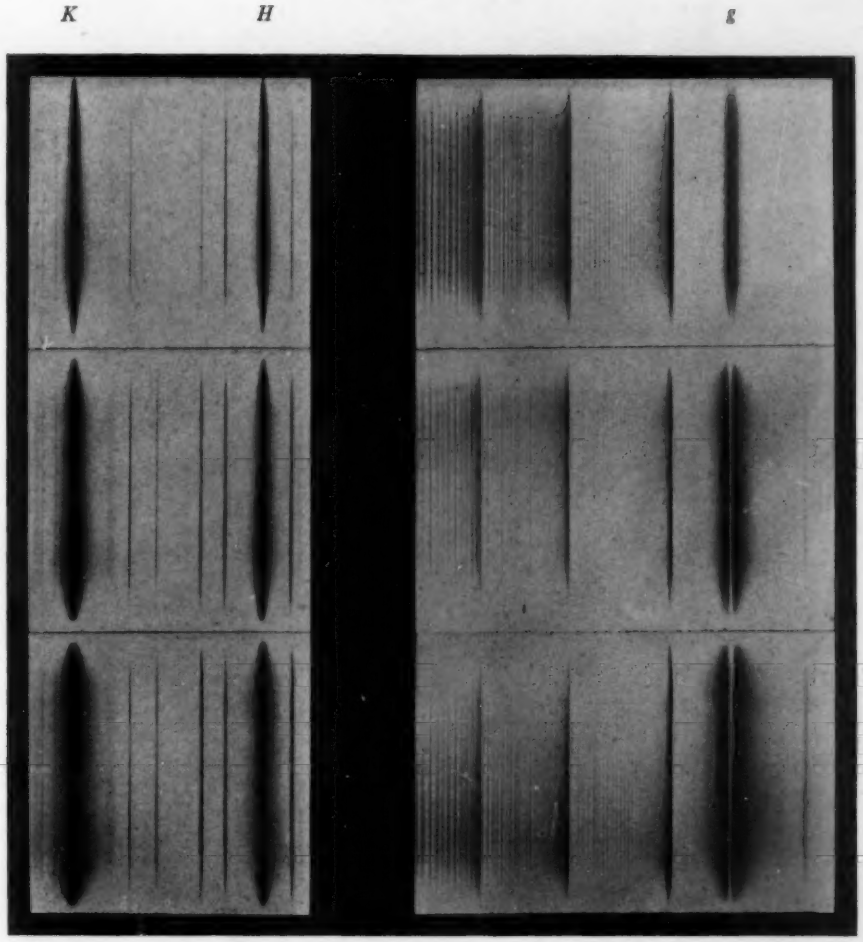
All the tubes were exposed at atmospheric pressure. To what extent electrical and chemical action took place in the vaporizing process is not known. It was assumed to be constant, and nothing in the appearance of the vapor while being generated would lead the observer to another opinion.

The H and K lines have always been observed together, K being always a trifle the wider, at least, if not the more intense. With faint H and K lines present, *g* was frequently missing. Whenever the background of continuous spectrum appeared strong, photographic reversal took place.

It is important to notice the changes in the lines upon approaching reversal. The line *g* is never as hard as the other two lines, even when these are very faint. As the vapor-density increases, it becomes more diffuse until reversal occurs. At this point K begins to lose its sharpness, and when it reverses H is almost ready to reverse.[1] In each case there seems to be a more or less gradual breaking up of the structure of the line before reversal takes place. When the three lines are reversed on the same plate, the wings of *g* flare out about a wide core which has clean sharp edges, while H and K flare very little, preserving their hard form tenaciously. The accompanying plate XIX shows the changes in these lines as the vapor-density is increased, the lowest spectrum corresponding to the greatest density.

[1] Concerning Dr. A. S. King's observations, in the *Astrophysical Journal*, **28**, 395, 1908, that H and K refused to show "self-reversal" in the furnace, the present results would seem to indicate that in his case the vapor of calcium was too rare to reverse these lines.

PLATE XIX



SUCCESSIVE REVERSAL OF CALCIUM LINES WITH INCREASING VAPOR-DENSITY

U of M

CONCLUSION

The lines *g*, λ 4227, and H, λ 3968, require for reversal, vapor-densities that are 1/7 and 3/2 as great respectively as that necessary for a reversal of K, λ 3934, when the vapor is observed at a temperature of 2500° C. and at atmospheric pressure.

It will be understood that the figures given above are approximations only. The difficulty of the problem and the limitations of the method alike render quite useless any claim to greater accuracy.

RYERSON PHYSICAL LABORATORY
THE UNIVERSITY OF CHICAGO
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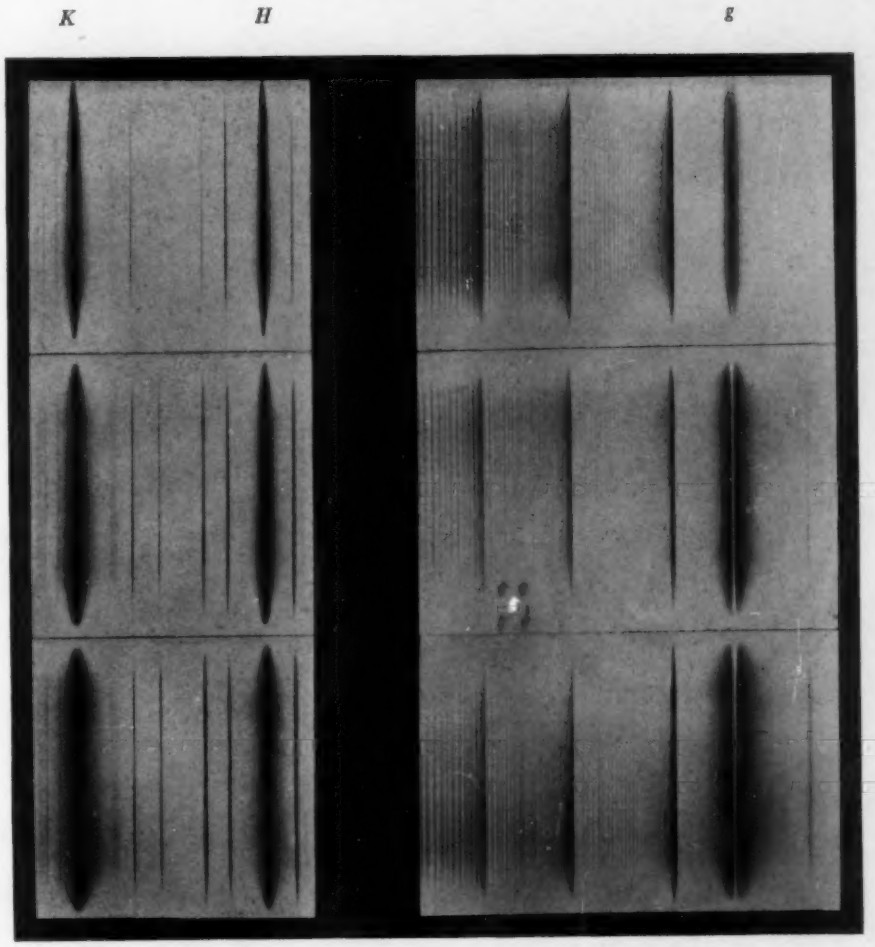

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THE UNIVERSITY OF CHICAGO
May 1911

NICKELED GLASS REFLECTORS FOR CELESTIAL PHOTOGRAPHY

By R. W. WOOD

The failure of silver to reflect well in the ultra-violet region, places a limit to the extent of the spectrum of a star or other celestial object obtained with a reflector of the ordinary type. As is well known, the spectrum of light which has penetrated the earth's atmosphere extends to about wave-length 2900. In the region around wave-length 3160 silver reflects only 4 per cent of the incident light; in other words, the silvered mirror is no more efficient in this region than unsilvered glass. For extending the work which I commenced two years ago on the ultra-violet photography of the moon, it became necessary to construct a long-focus mirror of large aperture capable of reflecting copiously the light transmitted through the silvered-quartz ray-filter. The great weight of a large mirror of speculum metal, and difficulties in its construction, with its liability to tarnish, led me to try to work out some method of depositing a film of some suitable metal upon a glass mirror.

Consulting Rubens' tables of the reflecting power of metals, it appeared that nickel would prove most suitable, its reflecting power throughout the entire spectrum being only a little less than that of speculum metal. So far as I was able to find, no method is known of depositing nickel chemically, except the process in which the gas nickel carbonyl is employed. This gas is extremely poisonous, and I understand that the metal films are bright only on the surface which is in contact with the glass. I accordingly determined to use this method only as a last resort, and commenced experiments at my East Hampton laboratory this summer on the electrolytic deposition. It was found that solutions with an acid reaction either frilled or completely removed the silver film, which was deposited chemically to serve as a substratum for the electrolytic film. I commenced work with copper, and found that if a dilute solution of the sulphate was converted into cuprammonium by the addition

PLATE XX

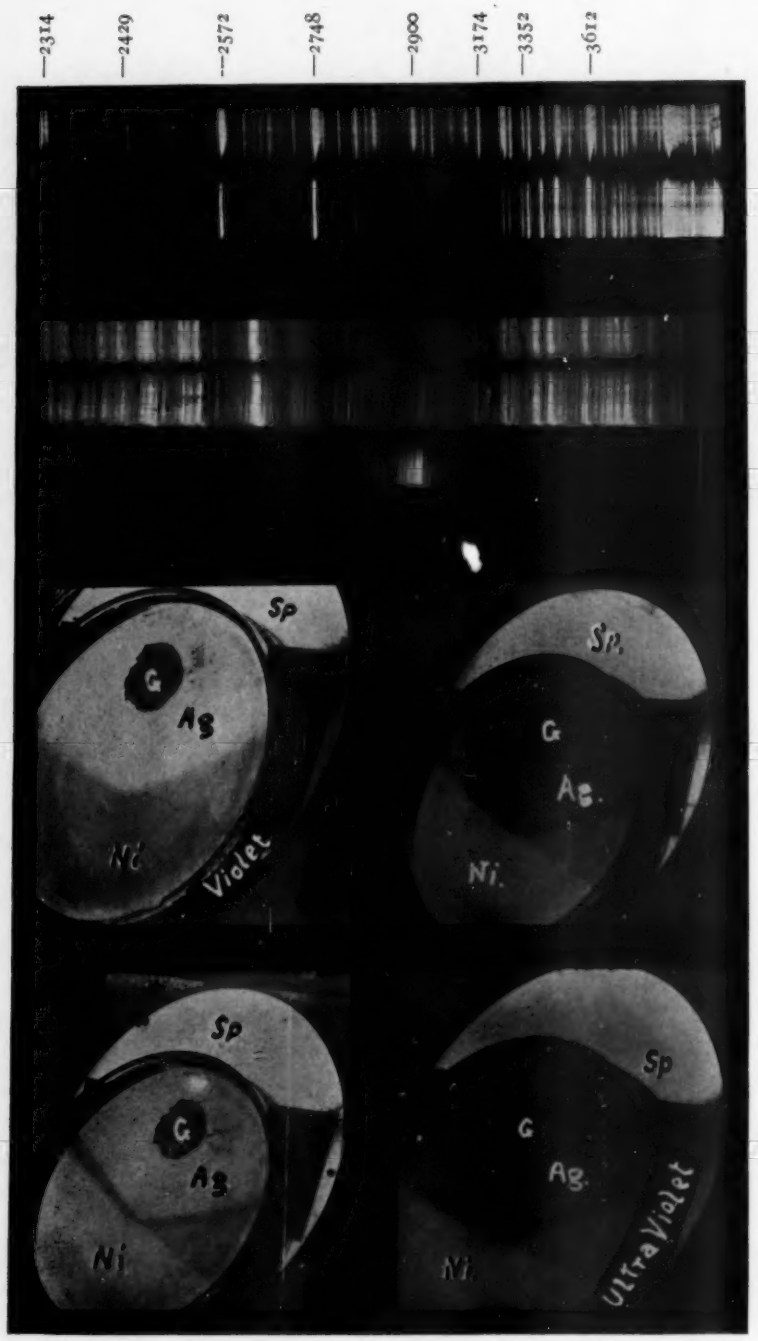

FIG. 1 *Cd* and *Sn* Spark

FIG. 2 *Fe* Spark

FIG. 3

REFLECTION FROM NICKEL AND SILVER FILMS

of ammonia, a beautiful deposit could be thrown down in about 30 seconds with two dry cells. I then tried the double sulphate of nickel and ammonia. This gave some trouble at first, but it finally yielded good deposits. The solution must be very dilute, however, say 10 grams to the liter, and it is a good plan to add a little ammonia to it, sufficient to give it a very slight odor. From one to three dry cells can be used, according to their condition and the size of the mirror to be coated. The silver substratum should be thin enough to show a deep blue color when examined by a strong light, and the nickel should be thrown down until it becomes opaque. Some experience will be necessary before good deposits are obtained. If the silver coating is too thin, the fall of potential along the surface from the point where the battery wire joins it is too great, and the deposit is not uniform. It is a good plan to practice with small glass plates until one becomes familiar with the difficulties, and learns how to avoid them.

A nickel anode of about the same area as the plate to be coated is joined to the carbon pole of the battery, though in the case of my 16-inch long-focus mirror, I used a 16-inch disk of copper; it makes little or no difference which is used, as very little metal is thrown out of solution. Before depositing the nickel I at first dried the silvered mirror, and polished off the light deposit of dust which usually clouds the surface. This is a bad plan, for it is almost impossible to avoid long scratches, which cut through and destroy the continuity of the conducting film and result in an erratic distribution of the current; if this is the case, the nickel deposits along one side of the scratch only, and the deposit is very streaky. If the current is too weak or if the current is applied for an insufficient time, one obtains a very thin, yellowish deposit which has a very low reflecting power in the ultra-violet, not much better than that of the silver. Continuing the process a little longer usually remedies this defect.

To study the reflecting power of the various films I reflected the light from the cadmium or iron spark from them into a quartz spectrograph, and obtained in this way spectrograms which enabled their behavior to be judged at once. On Plate XX, Fig. 1, we have two spectra of the spark of cadmium and tin, the upper, *a*, taken

with the light reflected from a nickel-on-glass film, the lower, *b*, with a silver film. As is at once apparent, one region of the spectrum in the latter photograph is wholly absent, and the entire ultra-violet below this region is relatively weak. Fig. 2 shows the same thing with the iron spark. Spectrogram *c* is made with direct light of the spark transmitted through the quartz-silver ray-filter, and shows that the region which the silver fails to reflect is copiously transmitted. Fig. 3 is extremely interesting and shows the reflecting power of glass, silver, speculum metal, and nickel for various regions of the spectrum.

A shallow glass dish was silvered on the inside. The lower portion of this deposit was then nickel plated, and a portion of the silver wiped from the upper portion with a bit of cotton moistened with very dilute nitric acid. The clear glass is marked "*G*" on the pictures, the nickel "*Ni*," the silver "*Ag*," and the plate of polished speculum metal, which stands behind the dish, "*Sp.*"

The mirrors were mounted near an open window at such an angle as to reflect the light of the sky to the camera, which was provided with a quartz lens. Photograph *A* was made with the quartz lens unsilvered, that is, chiefly through the action of the blue and violet rays. The silver and the speculum metal come out very bright, while the nickel is much darker, and the unsilvered glass spot *G* is practically black. *B* was made with the quartz lens coated with a film of silver of such thickness that a brightly lighted window appeared very deep blue through it. In this case we have a mixture of violet and ultra-violet in such proportion, that the silver, nickel, and speculum all reflect about equally well. The dark V-shaped band which separates the nickel area from the silver is the thin yellow deposit above referred to, which diminishes the reflecting power in the ultra-violet. No trace of this appears in the first picture. In the small triangular area below the "*Ag*" the nickel deposit is thinner than over the rest of the lower portion of the plate, and the reflecting power is less. Photograph *C* was made through a silver film of such thickness that a brightly lighted window was barely visible through it, and the sun appeared as a disk of a bluish-white color easily supported by the eye. This picture is made chiefly by the ultra-violet region between wave-lengths 3000

and 3200. The silver reflects only very little better than the spot of clear glass, and the speculum somewhat more strongly than the nickel. In the case of *D* the silver film was so thick that the sun could be seen only as a feebly luminous ball of a deep violet color, resembling the lighted window as seen with the thinner film. A tungsten lamp was absolutely invisible through it. This is the best thickness to employ for ultra-violet photography. The glass and silver both come out almost black, the nickel fairly bright, and the speculum a little brighter.

It appears to me that there is a wide field of investigation open in various branches of science, by means of photography with these obscure rays. I have found that many white substances are absolutely black in this ultra-violet light, zinc-oxide being a conspicuous example. White garden flowers have very different reflecting powers in this region of the spectrum. Common phlox comes out coal black, while white geraniums are very much lighter. The earlier photographs which I made were taken with a silver film which was much too thin, as the investigations of the past summer have shown, and it is impossible to predict what will be found in the pictures of the moon and planets which are soon to be made with a very thick silver film and the long-focus nickel mirror.

We will now take up the method of coating a large mirror with a deposit of nickel.

The mirror which I have nickeled for the ultra-violet photography of the moon and planets is 16 inches in diameter and has a focal length of 26 feet. It was made by Mr. John E. Mellish of Cottage Grove, Wis., and is of very good figure. The silver substratum was thrown down by the formaldehyde process[1] and the plate transferred to the nickel bath without drying, the surface, however, being first well washed under a rather heavy stream of water to remove as much of the sediment as possible. The deposition of the nickel was carried on in a cylindrical pan of galvanized iron, which I found in the local hardware store, the inner surface being coated with paraffine to prevent the solution from attacking the metal. The silver surface of the mirror was about an inch below the surface of the nickel solution, the anode consisting of a

[1] Wood, *Physical Optics*, revised edition.

circular disk of sheet copper of the same diameter as the mirror. A short piece of heavy brass tubing was soldered to the center of the copper disk to serve as a handle, and the wire from the battery was twisted around this. A V-shaped notch about an inch in depth was cut in the margin of the disk to facilitate making contact with the silver film. The other wire from the battery was attached to a slender rod of brass with a scrap of platinum-foil soldered to its lower end. This piece of foil was crumpled up so as to make contact at many points with the silver. The copper disk was carefully immersed in the solution and held so as to be everywhere nearly at the surface. This insured a uniform distance at every point between the anode and the silver film. The platinum electrode was now brought in contact with the silver film by passing the brass rod down through the V-shaped notch, and the current was allowed to flow for thirty seconds, after which the mirror was removed and held up to the light. The film was much darker than before but not quite opaque. Thirty seconds more finished the operation and gave a beautiful coat, which was washed, dried, and rubbed lightly with cotton to remove the slight film of sediment.

If the current-strength and density of solution have been right, the surface of the nickel, as soon as it is removed from the plating bath, should appear bright, though distinctly duller than the silver. It is a good plan to prepare a small glass plate with half of its surface silvered and the other half nickeled. In this way it is possible to judge of the quality of the nickel film, which should appear of about the same brightness as the silver film viewed through a nicol prism, which reduces the intensity of the reflected light by one-half. If the film appears yellow, or brownish, like smoked glass, something has gone wrong. If the silver peels from the glass or frills, the current has been too strong or it has been applied for too long a time. The coating which I have applied to my mirror is very nearly as bright as the surface of my 13-inch short-focus speculum. It may happen that the film appears to be all right until an attempt is made to clean it after it is dry, when it is found that the slightest rubbing with soft cotton strips the film from the glass. This is probably due to the fact that too much metal has been deposited.

As is well known, nickel deposited by electrolysis comes down under a tremendous tension. Even in technical work, where it is deposited on solid metal, it is apt to scale off if deposited too thickly. Under the circumstances, it is rather surprising that the silver film is able to stand up under the strain.

In addition to the advantage to be derived from the higher reflecting power in the ultra-violet, it is possible that the nickel film may prove more permanent in atmospheres contaminated with sulphur, though I have not had my mirror under observation for a sufficient length of time to be able to speak of its lasting qualities. One mirror which I prepared early in the summer appears to have changed, the surface having a crystalline appearance in spots, as if the two metals had alloyed. This mirror was made, however, before I had worked out the process to my satisfaction, the film being of the brownish color referred to above. The electrical process does not interfere with the figure, as the thickness of the layer deposited is but a very small fraction of the wave-length of light.

These experiments are a continuation of work commenced two years ago in which I was aided by a grant from the Elizabeth Thomson Science Fund.

EAST HAMPTON, LONG ISLAND
September 28, 1911

Reviews

An Introduction to the Theory of Optics. By Arthur Schuster. Second Edition, revised. London and New York: Longmans, Green & Co., 1910. $4.00.

This is the revised edition of Professor Schuster's well-known textbook on light, which first appeared seven years ago. The corrections called for by the latter were few in number; but the alterations have been numerous and are all helpful.

This book is different in principle and in detail from all others on the same subject, and on this account—apart from all other reasons—it makes delightful reading. It forms, in a way, the complement of an equally valuable book on Light, that by Professor R. W. Wood. One feels in reading each of these that one is in the hands of a master of the subject; each author gives both theory and experiment, but in Schuster's book theory is emphasized and made clear, while in Wood's it is the description of the experiments which one most enjoys.

It may not be amiss to call attention to the qualities which seem to the reviewer to distinguish this book from others bearing similar titles.

1. All theories are developed in such forms as to lend themselves to numerical calculations and verification.

This is an extraordinarily valuable feature; for students can with such ease lose sight of arithmetic when enjoying the calculus, and so often defer the searching test of a theory by the actual measurements of experiment.

In this connection reference may be made to the sections dealing with diffraction, optical instruments (especially the spectroscope), damping of vibrations of electrons, molecular scattering.

2. The various phenomena discussed, either on a large scale or in detail, are explained in terms both of quasi-homogeneous radiation and of ether impulses.

The homogeneous train of waves, as ordinarily written, is, as Schuster says, "a mathematical abstraction"; and so he introduces the idea of what he calls quasi-homogeneous radiations, such as are emitted by gases when made luminous by electrical excitation. To this idea he

gives mathematical expression, by considering the summation of a number of simple waves, whose amplitudes and wave-lengths vary slightly. White light may then be defined by making the range of wave-length sufficient.

But white light may be considered as an irregular succession of impulses; and as Schuster says: "We take the disturbance as it is and try to represent it analytically, and just as there are many ways of resolving a system of forces, so are there many ways of resolving the motion of light into elements with which we can deal analytically. The resolution by homogeneous waves is one, the resolution by impulses, another. Whenever the two methods seem to yield different results, a mistake has been made in their application."

The many uses Schuster makes of the impulse-treatment are most instructive; e.g., in dispersion, interference, Talbot's bands, etc.

3. Many questions, often unimportant ones, are discussed, in which every student is interested and yet which are passed over by many writers. (The difficulty of the explanation may be the reason for this omission.) A great part of the enjoyment one has in reading the book comes from these remarks which are often of the nature of *obiter dicta*.

Again, the consideration given the great features of the subject is always ample and always original. New points of view are emphasized, and the basis of all theories in mechanical principles is made clear. A few sections in which this is most noteworthy are the ones on emission of energy, the nature of light, absorption, etc.

In reading a book which one enjoys and finds inspiring, it is not one's duty to look for errors of commission or omission; this is certainly true of this book, in spite of the fact that Professor Schuster by a remark in the preface to this new edition tempts one to make a painstaking search for mistakes. Yet a reviewer must note what he sees. The name of Lewis Rutherfurd is given wrongly on p. 142; and it is doubtful if, by using metallic surfaces on which to rule gratings the life of the diamond point is prolonged. The hardness of speculum metal is much greater than is supposed. The great advantage in its use as a grating surface comes from the fact that in all probability grooves ruled in it reproduce most exactly the plane faces of the ruling-diamond; so that, from a knowledge of the angles of the latter, the shape of the groove may be known. This is contrary to the belief of Rowland and to the statement by Schuster on p. 111; but its justification may be found in a recent article in this journal (May 1911) by Anderson and Sparrow. In this same article there is a critical discussion of gratings, which points out

essential errors in that portion of Schuster's treatment given on pp. 122 and 123.

The reviewer is forced to confess that certain sections of the book have been understood with great difficulty: e.g., the discussion of absorption on pp. 270 and 271; the explanation of "the nature of light" on pp. 336–338; etc. One would not complain, if one did not believe that Professor Schuster might have made his treatment much simpler.

The order used in the arrangement of the subject-matter is not one to which we are accustomed; and the manner in which the student is referred to subsequent sections for vital discussions, are points of presentation or of pedagogy on which any teacher is privileged to differ with Professor Schuster. It may be, of course, that the training in English schools would make his scheme the natural one.

In conclusion, it should be stated clearly that no book on optics known to the reviewer is so eminently satisfactory, especially owing to the fact that it answers so many questions and reduces such a large part of the subject to the principles of mechanics.

JOSEPH S. AMES

JOHNS HOPKINS UNIVERSITY
June 1, 1911

The Principles and Methods of Geometrical Optics, Especially as Applied to the Theory of Optical Instruments. By JAMES P. C. SOUTHALL. New York: Macmillan, 1910. Pp. xxiv+626. $5.50.

This work aims to fill a recognized gap among scientific treatises in English. In extent and detail of treatment and in the scientific importance of the topics dealt with, it goes far beyond most other works in English; in fact, can be compared easily only with the recent book by Taylor, which is more restricted in scope and devoted primarily to the mathematical methods devised or used by its author, while the present volume is planned as a general treatise.

In the main the treatment is in the spirit of the recent German literature on the subject, from which indeed some portions appear rather as careful compilations than as independent presentation. Especial indebtedness is naturally owed and frankly acknowledged to the works of Czapski and of von Rohr and his colleagues. But the material from various sources has been thoughtfully fused into a coherent whole, and is supplemented by abundant and well-chosen bibliographic references. The notation in the formulas has evidently received unusual care, and

the reader is further aided by a complete list of the algebraic symbols, given in an appendix which accompanies the good index.

The early chapters discuss the scope and limitations of geometric optics as a physical theory, presenting its basic concepts and laws by an eclectic use of the alternative methods of rays, wave-surfaces, and optical paths. A brief study of refraction at a plane surface prepares for a long chapter on prisms, which deals chiefly, however, with rays in a meridian plane; the treatment of oblique rays seems unduly meager, and the problem of the curvature of spectrum lines is dismissed with a list of references.

The elementary or paraxial theory of spherical surfaces and of lenses is given first in the older form of an approximation based on the law of refraction for rays of small obliquity and is then exhibited as the ideal implied in the purely geometric theory of optical imagery as developed principally by Abbe. To this modern fundamental theory of the general properties of optical collineations is devoted a full and excellent chapter, which will doubtless be especially welcome to readers to whom the German publications are not accessible.

Following this are given various sets of exact trigonometric formulas for rays following the physical law of refraction, together with a study of astigmatic pencils of arbitrary inclination. The remainder of the volume, except for the closing chapter on aperture, field, and illumination, presents the theory of the various spherical and chromatic aberrations of the first order, mainly in the form due to Petzval and Seidel, aided by some of the concepts and notations introduced by Abbe. For the pure theory the treatment here would be hard to improve.

Some shortcomings might be noted, partly in matters of physical optics, but more particularly in matters touching on the practical side, the detailed work of the maker or user of optical instruments. The formulas are not always adapted for computation and the numerical examples are meager and rather trivial. The list of Jena glasses is unfortunate in its selections, since half of the types given are perishable and no longer made, while the remainder are hardly representative of the optical materials available at the present time. But the book excludes from its plan a systematic discussion of special types of optical constructions and is therefore judged best simply as a repository of the general basic theory. As such it can hardly be said to have a superior in any language, and must surely be ranked as a standard work.

A. C. L.

Books Received

CH. ANDRÉ: *Les planètes et leurs origines.* Pp. 285, Pls. 3, Figs. 94. Paris: Gauthier-Villars, 1909. Fr. 10.

C. FLAMMARION: *La planète Mars et ses conditions d'habitabilité.* Pp. 608, 580 dessins telescopiques, et 23 cartes. Paris: Gauthier-Villars, 1892. Broché, fr. 12; cartonné, fr. 15.

PERCIVAL LOWELL: *Mars as the Abode of Life.* Pp. 288, Pls. 8, Figs. 65. New York: Macmillan, 1908. $2.50 net.

A. MARCUSE: *Astronomische Ortsbestimmung im Ballon.* Pp. 67, 3 Karten, und 3 Textbilder. Berlin: G. Reimer, 1909.

W. HERZ: *Les bases physico-chimiques de la chimie analytique.* Pp. 167, Figs. 13. Paris: Gauthier-Villars, 1909. Cartonné, fr. 5.

G. LE BON: *The Evolution of Forces.* Pp. 388, Frontispiece, and Figs. 42. New York: D. Appleton & Co., 1908.

A. BERBERICH: *Astronomischer Jahresbericht,* Bd. XI, Die Litteratur des Jahres 1909. Pp. 726. Berlin: G. Reimer, 1910.

C. G. KNOTT: *The Physics of Earthquake Phenomena.* Pp. 283, Figs. 43. Oxford: The Clarendon Press, 1908. 14*s.* net.

E. BELOT: *Essai de cosmogonie tourbillonnaire.* Pp. xi+280, Figs. 52. Paris: Gauthier-Villars, 1911. Fr. 10.

C. FLAMMARION: *Annuaire astronomique et météorologique pour 1911.* Pp. 332, Figs. 92. Paris: E. Flammarion, 1910. Fr. 1.50.

B. WANACH: *Tafel der Werte* $\frac{a \cdot b}{a+b}$. Pp. 25. Berlin: B. G. Teubner, 1910. M. 1.20.

F. K. MCCLEAN: *Report of the Solar Eclipse Expedition to Port Davey, Tasmania, May, 1910.* Pp. 42, Pls. 35. London: Richard Clay & Sons, Ltd.

M. VON ROHR: *Die optischen Instrumente.* Zweite Auflage. Pp. 140, Figs. 88. Berlin: B. G. Teubner, 1911. M. 1.25.

WILHELM TRABERT: *Lehrbuch der kosmischen Physik.* Pp. 662, Figs. 149. Berlin: B. G. Teubner, 1911. M. 20; geb. M. 22.

ROBERT W. WOOD: *Physical Optics.* New and revised edition. Pp. 705, Figs. 399. New York: Macmillan, 1911. $5.25 net.

GILBERT T. WALKER: *Outlines of the Theory of Electromagnetism.* Pp. 52. Cambridge: The University Press, 1910. 3*s.* net.

A. G. WHYTE: *Electricity in Locomotion.* Pp. 143, Figs. 18. Cambridge: The University Press, 1911. 1*s.* net.

Transactions of the International Union for Co-operation in Solar Research, Vol. 3. Pp. 231, Figs. 8. Manchester: The University Press, 1911. 7*s.* 6*d.* net.

M. VON ROHR: *Die Brille als optisches Instrument.* Pp. 158, Figs. 48. Leipzig: Wilhelm Engelmann, 1911.

W. L. MOORE: *Descriptive Meteorology.* Pp. 344, 45 charts, Figs. 81. New York: D. Appleton & Co., 1910. $3.00 net.

ARTHUR SCHUSTER: *The Progress of Physics during 33 Years (1875–1908).* Pp. 164, with portrait of Maxwell. Figs. 10. Cambridge: The University Press, 1911. 3*s.* 6*d.*

CH. HENRIONNET: *Petit traité d'astronomie pratique.* Paris: Gauthier-Villars, 1911. Fr. 1.75.

WILLIAM TYLER OLCOTT: *Star Lore of All Ages.* Pp. 453, with 114 illustrations. New York: Putnam, 1911. $3.50 net.

SIR ALGERNON F. R. DE HORSEY: *Draysonia.* Pp. 76. London: Longmans, Green & Co., 1911. 3*s.* 6*d.* net.

ERRATA

Vol. **34,** September 1911, in Professor Stebbins' article on "A New Bright Variable Star, *β Aurigae*":

Page 119, in (2), *for* m_1+etc., *read* m_1−etc.

Page 125, twelfth line from bottom, in coefficient of δi, *for* sin 2η, *read* sin η. This equation should be numbered (5).

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DECEMBER 1911

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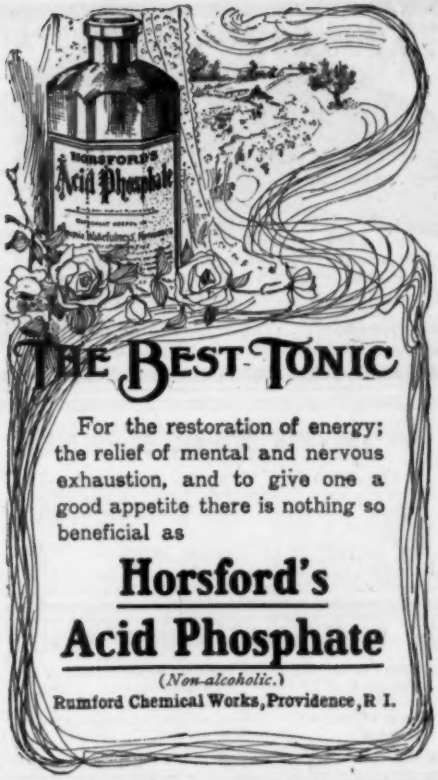

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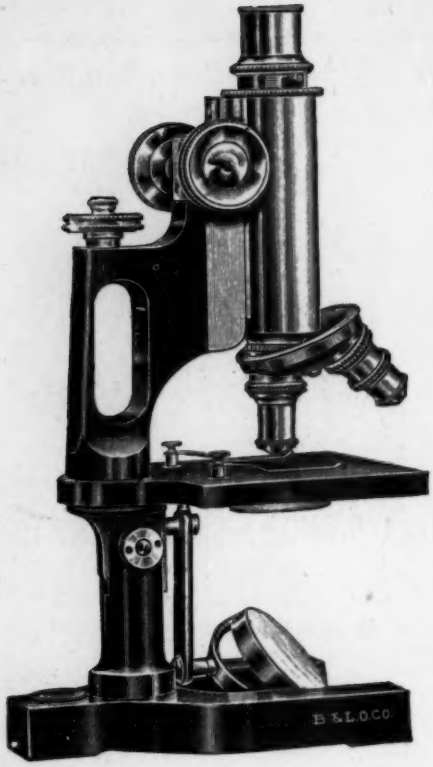

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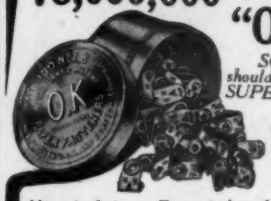

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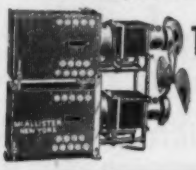

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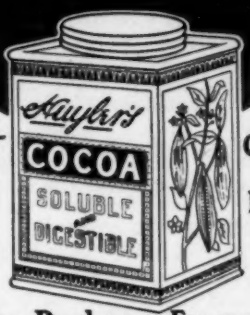

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